Monday 7 January 2019

Mathematics B
Paper 1

You must have: Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Instructions

• Use black ink or ball-point pen.
• Fill in the boxes at the top of this page with your name, centre number and candidate number.
• Answer all questions.
• Answer the questions in the spaces provided – there may be more space than you need.
• Calculators may be used.

Information

• The total mark for this paper is 100.
• The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Check your answers if you have time at the end.
• Without sufficient working, correct answers may be awarded no marks.
Answer ALL TWENTY SEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 A shop sells a tablet computer for $230
   The shop increases the price of the tablet computer by 5%
   Calculate the price of the tablet computer after this price increase.

$ ..................................................  

(Total for Question 1 is 2 marks)

2 The $n$th term of a sequence is given by $5n^2 - 2$
   Find the first 3 terms of this sequence.

 .............................................  , .............................................  , .............................................  

(Total for Question 2 is 2 marks)
3. The lengths of the sides of a triangle are in the ratios 4:5:6
The perimeter of the triangle is 80 cm.

Find the length of the longest side of the triangle.

..................................................................... cm
(Total for Question 3 is 2 marks)

4. Factorise fully $9x^2y + 12xy^3z$

.....................................................................
(Total for Question 4 is 2 marks)

5. Make $x$ the subject of $y = \frac{8(x + 1)}{w}$

.....................................................................
(Total for Question 5 is 2 marks)
6 Simplify fully \( \frac{(2xy^2)^3}{4xy^2} \)

(Total for Question 6 is 2 marks)

7

The diagram shows a curve drawn on a grid.

By drawing a suitable straight line on the grid, find an estimate, to one decimal place, of the gradient of the curve at the point when \( x = 1 \)

(Total for Question 7 is 2 marks)
8  (a) Solve the inequality \( 5x + 4 < 20 - 3x \)

(b) Represent, on the number line below, your solution of the inequality in part (a).

![Number Line Diagram](image)

(Total for Question 8 is 3 marks)

9  The points \( O, A, B \) and \( C \) are such that \( \overrightarrow{OA} = \left( \begin{array}{c} -2 \\ 4 \end{array} \right), \overrightarrow{OB} = \left( \begin{array}{c} 5 \\ -3 \end{array} \right) \) and \( \overrightarrow{AC} = \left( \begin{array}{c} -1 \\ -1 \end{array} \right) \)

Find \( |\overrightarrow{BC}| \)

\[ |\overrightarrow{BC}| = \]
10 (a) Write 340 000 000 in standard form.

(b) Calculate, giving your answer in standard form, \((2 \times 10^{99}) \div (5 \times 10^{110})\)

(Total for Question 10 is 3 marks)

11 Given that, for all values of \(x\),

\[2x^2 - 3x + 21 = a(x - b)^2 + c\]

find the value of \(a\), the value of \(b\) and the value of \(c\).

\[a = \ldots\]
\[b = \ldots\]
\[c = \ldots\]

(Total for Question 11 is 3 marks)
12

\[ AOC \text{ and } BOD \text{ are diameters of a circle, centre } O. \]

Prove that triangle \( ABC \) and triangle \( DCB \) are congruent.
The diagram shows a sector $OAB$ of a circle, centre $O$.

$OA = OB = 3.6$ cm
Angle $AOB = 85^\circ$

Calculate the perimeter, in cm to 3 significant figures, of the sector $OAB$. 

\[
\text{Perimeter} = \text{arc length} + 2 \times \text{radius} = \frac{85 \times 3.6}{180} + 2 \times 3.6 \\
= 0.85 + 7.2 \\
= 8.05 \\
\text{cm}
\]

(Total for Question 13 is 3 marks)
In the diagram, $ABC$ is a triangle with point $D$ on $AB$ and point $E$ on $AC$ such that $DE$ is parallel to $BC$.

$\angle ABC = 40^\circ \quad \angle AED = 110^\circ \quad \angle BAC = x^\circ$

Find the value of $x$.

Give reasons for each stage of your working.

$x = \boxed{\text{..................................................................}}$

(Total for Question 14 is 3 marks)
15 A car travelled at an average speed of 96 km/h for \(1\frac{1}{2}\) hours and then travelled at an average speed of 56 km/h for \(2\frac{1}{2}\) hours.

Calculate the average speed of the car for the whole journey.

\[\text{km/h} \]

(Total for Question 15 is 4 marks)
The diagram shows the position of a rock $R$, a boat $B$ and a lighthouse $L$. The bearing of $L$ from $R$ is $156^\circ$.

(a) Calculate the bearing of $R$ from $L$.

Given that the bearing of $B$ from $R$ is $072^\circ$ and that $RB = RL$,

(b) calculate the bearing of $L$ from $B$.

(Total for Question 16 is 4 marks)
The diagram shows a shape $ABCDE$ made from a rectangle $R$ and a semicircle $S$. The side $DB$ of the rectangle is the diameter of the semicircle.

Given that $AB = 9\text{ cm}$ and that the area of $S = 2\pi \text{ cm}^2$,

find the area of $R$.

\[
\text{cm}^2
\]

(Total for Question 17 is 4 marks)
### Question 18

Let \( A = \begin{pmatrix}-2 & -4 \\ 1 & 3\end{pmatrix} \) and \( BA = \begin{pmatrix}2 & -8 \\ 1 & 2\end{pmatrix} \).

Find the matrix \( B \).

(Total for Question 18 is 4 marks)
A solid cone has a mass of 1200 g, to 2 significant figures.
The area of the base of the cone is 38.5 cm$^2$, to 1 decimal place.
The height of the cone is 5.0 cm, to 2 significant figures.

Given that

\[
\text{density} = \frac{\text{mass}}{\text{volume}}
\]

calculate the upper bound of the density of the cone.
Give your answer to 3 significant figures.

\[
\text{......................... g/cm}^3
\]
20

ABCD and PQRS are two similar quadrilaterals.
The side AD of length \((2x + 4)\) cm is the longest side of quadrilateral \(ABCD\).
The side PS of length \(4(x^2 - 4)\) cm is the longest side of quadrilateral \(PQRS\).
The area of the quadrilateral \(ABCD\) is \(10\) cm\(^2\).

Show that the area, in cm\(^2\), of the quadrilateral \(PQRS\) is \(a(x - b)^2\)
where \(a\) and \(b\) are integers to be found.

(Total for Question 20 is 4 marks)
21 The lengths of cod caught by fishermen in a small fishing boat last month were measured. The incomplete table and histogram below give information about the length of each cod caught last month.

<table>
<thead>
<tr>
<th>Length (l cm)</th>
<th>Number of cod</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 &lt; l \leq 25$</td>
<td>30</td>
</tr>
<tr>
<td>$25 &lt; l \leq 40$</td>
<td>75</td>
</tr>
<tr>
<td>$40 &lt; l \leq 50$</td>
<td>100</td>
</tr>
<tr>
<td>$50 &lt; l \leq 60$</td>
<td>155</td>
</tr>
<tr>
<td>$60 &lt; l \leq 80$</td>
<td></td>
</tr>
<tr>
<td>$80 &lt; l \leq 100$</td>
<td>20</td>
</tr>
</tbody>
</table>

(a) Complete the table and the histogram.
Cod that have a length less than 35 cm are returned to the sea.

(b) Calculate an estimate, to one decimal place, of the percentage of cod caught that were returned to the sea from the boat last month.

.......................................................... %

(Total for Question 21 is 5 marks)

22 The width of a rectangle is $x$ metres.
The length of the rectangle is 7 m longer than the width of the rectangle.

Find the set of values of $x$ for which the area of the rectangle is less than 44 m$^2$
Show clear algebraic working.

..........................................................
Josh asks the students in his form how many hours they played sport last week.

The table shows information about his results.

<table>
<thead>
<tr>
<th>Number of hours ((t))</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 \leq t &lt; 1)</td>
<td>9</td>
</tr>
<tr>
<td>(1 \leq t &lt; 2)</td>
<td>8</td>
</tr>
<tr>
<td>(2 \leq t &lt; 3)</td>
<td>5</td>
</tr>
<tr>
<td>(3 \leq t &lt; 4)</td>
<td>7</td>
</tr>
<tr>
<td>(4 \leq t &lt; 6)</td>
<td>3</td>
</tr>
<tr>
<td>(6 \text{ or more})</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) Find the class interval that contains the median number of hours.

........................................................................
(1)

(b) Calculate an estimate, to 3 significant figures, for the mean number of hours that the students in Josh’s form played sport last week.

........................................................................................................................................ hours
(4)

(Total for Question 23 is 5 marks)
24 The function \( f \) is defined by \( f : x \mapsto 4 + \frac{3}{x} \quad x > 0 \)

(a) Find \( f(2) \)

(b) Find the range of \( f \).

(c) Express the inverse function \( f^{-1} \) in the form \( f^{-1} : x \mapsto \ldots \)

The function \( g \) is defined by \( g : x \mapsto 4x - 5 \)

(d) Express the composite function \( fg \) in the form \( fg(x) = \ldots \)

(Total for Question 24 is 6 marks)
The diagram shows a pyramid with triangular base $OAC$. The edges $OA$, $OB$ and $OC$ of the pyramid are perpendicular to each other.

$OA = 7\text{ cm} \quad OB = 12\text{ cm} \quad OC = 5\text{ cm}$

(a) Calculate the volume, in cm$^3$, of the pyramid.

\[
\frac{1}{3} \times \text{base} \times \text{height} \]

\[
\frac{1}{3} \times \frac{1}{2} \times 7 \times 12 \times 5 = \frac{1}{3} \times 21 \times 12 \times 5 = 210 \text{ cm}^3
\]

\[210 \text{ cm}^3\]
(b) Calculate the area, in cm² to 3 significant figures, of triangle $ABC$. 

\[ \text{\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots cm}^2 \] 

(6) 

(Total for Question 25 is 8 marks)
26 Solve the simultaneous equations

\[ x^2 = 10 - y^2 \]
\[ x + 2y = 5 \]

Show clear algebraic working.
27 A curve has equation \( y = x^3 - 4x^2 + 2x \)

Find the \( x \) coordinate of each of the points on the curve at which the gradient of the tangent to the curve is 2

(Total for Question 27 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS