You must have: Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Instructions

• Use black ink or ball-point pen.
• Fill in the boxes at the top of this page with your name, centre number and candidate number.
• Answer all questions.
• Answer the questions in the spaces provided – there may be more space than you need.
• Calculators may be used.

Information

• The total mark for this paper is 100.
• The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Check your answers if you have time at the end.
• Without sufficient working, correct answers may be awarded no marks.
Answer ALL ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. There are $x$ large houses and $y$ small houses in a street.
   
   Each large house has 3 cars parked outside of it and each small house has 2 cars parked outside of it.
   
   The total number of cars parked outside all of the houses in the street is 80

   (a) Write down an equation in $x$ and $y$ for this information.

   Each large house has 8 windows and each small house has 5 windows.
   The houses in the street have a total of 204 windows.

   (b) Write down another equation in $x$ and $y$ for this information.

   (c) Hence find the number of large houses in the street and the number of small houses in the street.
2 Solve the equation \( \frac{5x + 3}{x - 2} = \frac{x - 2}{x + 1} \).

Give your solutions to 3 significant figures.

Show your working clearly.

\[
\text{Solutions of } ax^2 + bx + c = 0 \text{ are } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
3 \[ A = \begin{pmatrix} \frac{1}{2}y & 0 \\ 0 & 2x \end{pmatrix} \] where \( x \neq 0 \) and \( y \neq 0 \)

(a) Write down the inverse matrix, \( A^{-1} \), of the matrix \( A \).

(b) Hence, or otherwise, find the value of \( x \) and the value of \( y \) such that

\[
\begin{pmatrix} \frac{1}{2}y & 0 \\ 0 & 2x \end{pmatrix} \begin{pmatrix} y - 2 \\ 4 \end{pmatrix} = \begin{pmatrix} y \\ x^4 \end{pmatrix}
\]
The members of a sports club can play cricket (C), football (F) and golf (G).

The members of the club were asked which of these sports they play.

Here is some information about the numbers of members who play these sports.

\[ n(C \cap F \cap G) = 6 \]
\[ n(C \cap F \cap G') = 5 \]
\[ n(G \cap [C \cup F]') = 15 \]
\[ n(C) = 30 \]

The incomplete Venn diagram, where \( x \) is a positive integer, shows some other information about the numbers of members who play these sports.

(a) Use all the given information to complete the Venn diagram.

(b) Given that \( n([F \cup G] \cap C') = 45 \), find the value of \( x \).

A member of the sports club is chosen at random.

(c) Using your answers to parts (a) and (b), find the probability that this member

(i) plays only cricket,

(ii) plays only two sports.
Question 4 continued

(Total for Question 4 is 9 marks)
A shopkeeper pays a total of $570 to buy 300 identical items.

The shopkeeper sells 200 of these items. The selling price of each of these 200 items is such that the shopkeeper makes a profit of 20% on what he paid for each item.

The shopkeeper then reduces this selling price by 25% and he sells the remaining 100 items at this reduced price.

Calculate the total profit made by the shopkeeper in selling all 300 items.
6 John and Peter play a game with a pack of 26 white cards. The pack has 13 cards with a blue spot in the middle of one side of the card and 13 cards with a red spot in the middle of one side of the card.

John and Peter take it in turns to pick at random a card from the pack. The card is not returned to the pack.

The **winner of the game is the first person to pick a card with a blue spot**.

John picks at random a card from the pack and does not return the card to the pack.

(a) Write down the probability that John wins the game with his first card.

If John does not win the game with his first card, then Peter picks at random a card from the pack and does not return the card to the pack.

If Peter picks a card with a blue spot then he wins the game.

If Peter does not win the game with his first card, then John picks at random a second card from the pack and does not return the card to the pack.

If John does not win the game with his second card, then Peter picks at random a second card from the pack.

If Peter does not win the game with his second card the game stops and the result is a draw.

The incomplete probability tree diagram, on page 13, represents a game in which John and Peter can pick at most two cards each.

(b) Complete the probability tree diagram for this game.

(c) Work out the probability that Peter wins the game with his first card.

(d) Work out the probability that Peter wins the game.
Question 6 continued

John wins

John does not win

Peter wins

Peter does not win

John wins

Peter wins

John does not win

Peter wins

Peter does not win
Figure 1 shows a triangular based pyramid $ABCD$ in which the edge $BD$ is perpendicular to the edge $BC$ of the pyramid.

In $\triangle ABC$, $AC = 20\,\text{cm}$, $\angle BAC = 30^\circ$ and $\angle ABC = 100^\circ$

(a) Calculate the length, in cm to 3 significant figures, of $BC$.

In $\triangle DCB$, $\angle DCB = 40^\circ$

(b) Calculate the length, in cm to 3 significant figures, of $CD$.

Given that $AD = 12\,\text{cm}$, calculate, to the nearest whole number,

(c) the size, in degrees, of $\angle ADC$,

(d) the area, in cm$^2$, of $\triangle ADC$.

\[
\begin{align*}
\text{Sine rule: } \frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\
\text{Cosine rule: } a^2 &= b^2 + c^2 - 2bc \cos A \\
\text{Area of triangle} &= \frac{1}{2}bc \sin A
\end{align*}
\]
The points $(-3, -2)$, $(-2, 0)$ and $(-1, -1)$ are the vertices of triangle $A$.

(a) On the grid, draw and label triangle $A$.

Triangle $A$ is transformed to triangle $B$ under the transformation with matrix $P$ where

$$
P = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$$

(b) On the grid, draw and label triangle $B$.

Triangle $B$ is transformed to triangle $C$ under the translation $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

(c) On the grid, draw and label triangle $C$.

Triangle $C$ is transformed to triangle $D$ under the transformation with matrix $Q$ where

$$
Q = \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$$

(d) On the grid, draw and label triangle $D$.

(e) Describe fully the single transformation which maps triangle $A$ onto triangle $D$.
Question 8 continued

Turn over for a spare grid if you need to redraw your triangles.
Figure 2 shows the triangle $OCB$ in which $\vec{OC} = 2\mathbf{a}$ and $\vec{OB} = 12\mathbf{c}$ 

(a) Find $\vec{CB}$, giving your answer in terms of $\mathbf{a}$ and $\mathbf{c}$.  

$A$ is the midpoint of $OC$ and $D$ is the point on $BC$ such that $AD$ is parallel to $OB$ and triangles $CAD$ and $COB$ are similar.

(b) Explain why $\frac{AC}{OC} = \frac{DC}{BC} = \frac{AD}{OB} = \frac{1}{2}$ 

(c) Express in terms of $\mathbf{a}$ or $\mathbf{c}$ or $\mathbf{a}$ and $\mathbf{c}$, 

(i) $\vec{AD}$  

(ii) $\vec{OD}$ 

The point $E$ on $AD$ is such that $AE : ED = 1 : m$, where $m$ is an integer. 
The point $F$ on $OB$ is such that $CEF$ is a straight line.

(d) Show that $\vec{FD} = \mathbf{a} + 6\mathbf{c} - \frac{12}{m+1}\mathbf{c}$ 

Given that $\vec{FD} = \mathbf{a} + 3\mathbf{c}$  

(e) find the value of $m$. 

Given that the area of triangle $ACD$ is $10\text{ cm}^2$  

(f) calculate the area, in cm$^2$, of triangle $FCB$. 


10  \[ y = -\frac{x^3}{10} + 2x - \frac{1}{x^2} \]

(a) Complete the table of values for  \( y = -\frac{x^3}{10} + 2x - \frac{1}{x^2} \) giving your values of \( y \) to 1 decimal place.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>−3.0</td>
<td>0.9</td>
<td>3.0</td>
<td></td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) On the grid, draw the graph of  \( y = -\frac{x^3}{10} + 2x - \frac{1}{x^2} \) for 0.5 ≤ \( x \) ≤ 5

(c) Use your graph to find an estimate of the maximum value of \( y \), to 1 decimal place, for values of  \( x \) in 0.5 ≤ \( x \) ≤ 5

(d) Use your graph to find the range of values of  \( x \), for which  \( -\frac{x^3}{10} + 2x - \frac{1}{x^2} > 0 \) in 0.5 ≤ \( x \) ≤ 5

Give your values to 1 decimal place.
Question 10 continued

Turn over for a spare grid if you need to redraw your graph.
11 A solid is made by fixing a solid hemisphere of radius $r$ cm on the flat circular top face of a solid cylinder of radius $r$ cm and height $h$ cm. The centre of the hemisphere coincides with the centre of the flat circular top face of the cylinder as shown in Figure 3.

![Diagram NOT accurately drawn](image)

Figure 3

Given that the total external surface area of the solid is $S$ cm$^2$

(a) show that $S = \pi r(3r + 2h)$

(b) show that $h = \frac{25}{\pi r} - \frac{3r}{2}$

The total volume of the solid is $V$ cm$^3$

(c) Show that $V = 25r - \frac{5\pi r^3}{6}$

(d) Using calculus, find the value of $r$ for which the volume of the solid is a maximum.

\[
\begin{align*}
\text{Area of circle} &= \pi r^2 \\
\text{Curved surface area of a right circular cylinder} &= 2\pi rh \\
\text{Surface area of sphere} &= 4\pi r^2 \\
\text{Volume of sphere} &= \frac{4}{3} \pi r^3
\end{align*}
\]