Mark Scheme (Results)

Summer 2019

Pearson Edexcel International Advanced Level
In Mathematics
Mechanics M3 (WME03)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>'M' marks</u>
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

<u>'A' marks</u>
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

<u>'B' marks</u>
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

  Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton’s Second Law (Equation of Motion)

  NEL Newton’s Experimental Law (Newton’s Law of Impact)

  HL Hooke’s Law

SHM   Simple harmonic motion

PCLM   Principle of conservation of linear momentum

RHS, LHS   Right hand side, left hand side.

Note: if they don't have $\frac{1}{4}$ (or equivalent) on second line LHS, they will get the correct result, but should lose all A marks.

Question Number	Scheme	Marks
1	$\frac{1}{2}v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{k}{x^2}$	M1
	$\frac{1}{4}v^2 = \frac{k}{x} \quad (+c)$	M1A1
	$t = 0 \;\; x = 2 \;\; v = 5 \;\Rightarrow \frac{25}{4} = \frac{k}{2} + c$	dM1
	$x = 5 \;\; v = 4 \;\Rightarrow \frac{4^2}{4} = \frac{k}{5} + c$	A1
	$k = \frac{15}{2} \quad c = \frac{5}{2}$	dM1A1
	$v^2 = \frac{30}{x} + 10$	A1 (8)
		[8]

M1	For equation of motion with acceleration in the form $v\frac{\mathrm{d}v}{\mathrm{d}x}$ oe. Must have used a mass.
M1	Separating the variables and integrating. They must have had $v\frac{\mathrm{d}v}{\mathrm{d}x}$ oe
A1	Correct integration, constant not needed. They must have started with a minus sign. Note that k is positive since $\frac{k}{x^2}$ is a magnitude.
dM1	Using one of the boundary conditions to form an equation in k and c. Dependent on the first M1.
A1	Both equations correct
dM1	Solving their equations to find values for k and c. Dependent on the previous M mark.
A1	Correct values for k and c
A1	Correct expression for v^2
ALT	-if they use definite integration to find k and then indefinite to find c
	First 3 marks as per the main scheme, then
dM1	$\int_5^4 \frac{1}{2}v\,dv = -\int_2^5 \frac{k}{x^2}\,dx$
A1	$k = \frac{15}{2}$ from correct working.
dM1	Using indefinite integration and one set of limits to find c.
A1	$c = \frac{5}{2}$ o.e. from correct working.
A1	Correct expression for v^2

Question Number	Scheme	Marks
2	EPE $\dfrac{2mg(0.25l)^2}{2l}$ or $\dfrac{2mg(0.1l)^2}{2l}$	B1 either
	WD against friction: $0.35l\mu mg$	B1
	$\dfrac{2mg(0.25l)^2}{2l}-\dfrac{2mg(0.1l)^2}{2l}=0.35l\mu mg$	M1A1
	$\mu=0.15$ $\left(\text{or } \dfrac{3}{20}\right)$	M1A1 (6)
		[6]

B1	A correct expression for EPE, either at the start or end of the motion.
B1	Correct expression for Work Done against friction
M1	Attempt at an energy equation with a **difference** between 2 EPE terms and a WD . EPE must be of form $\dfrac{\lambda x^2}{kl}$, $k=2$ or 1
A1	Correct equation
M1	Solving to find μ. Independent, but must have 3 terms, but condone EPE added.
A1	$\mu=0.15$ $\left(\text{or } \dfrac{3}{20}\right)$ o.e. (g cancels)

Question Number	Scheme	Marks
3.	Vol of cylinder $=8\pi r^2 h$ Vol cut away $=\frac{1}{3}\pi r^2 h$	
	Cylinder / part cut away / conical shell / container	
	mass: M ; $(\pm)\frac{1}{24}M$; $\frac{1}{12}M$; $\frac{25}{24}M$	M1A1
	dist from O: h ; $\frac{7}{4}h$; $5h$; $\bar{x}$	B1
	$Mh-\frac{7}{96}Mh+\frac{5}{12}Mh=\frac{25}{24}M\bar{x}$	M1A1
	$\bar{x}=\frac{129}{100}h$ or $1.29h$	A1
		[6]

M1	Attempt at mass ratios. Must have 4 masses. M may be consistently omitted. Condone omission of 1/3 in volume of cone. Must be using the given masses, not volumes.
A1	Correct mass ratios. Sign not needed for cut away part.
B1	Correct distances (from a consistent point – see below)
M1	Attempt at moments equation. Condone missing minus. Accept as part of a vector equation.
A1	Correct equation.
A1	$\bar{x}=\frac{129}{100}h$ or $1.29h$

ALT distances

	Cylinder	Cut away	Shell
Vertex	0	$\frac{3}{4}h$	$4h$
Top of cylinder	$(-)h$	$(-)\frac{1}{4}h$	$3h$
Top of cone	$6h$	$\frac{21}{4}h$	$2h$

Note: If completing in 2 stages, centre of mass with cut-away cylinder without shell is $\frac{89}{92}h$.

Marks can only be awarded when they give a complete method, including the conical shell.

Special Case: If they first find the CoM of cut-away cylinder, then use M for its mass, award max M0A0 B1 M1A1 A0. Their equation must be $M\times\frac{89}{92}h+\frac{M}{12}\times 5h=\frac{13M}{12}\bar{x}$

4	Any correct trig function	B1
	$T_b + T_a \cos\theta = 8ma\omega^2$	M1A1A1
	$T_a \sin\theta = mg$	M1A1
	$T_b = 8ma\omega^2 - \dfrac{mg}{\sin\theta} \times \cos\theta$	dM1
	$8ma\omega^2 - \dfrac{mg}{\sin\theta}\cos\theta \le \dfrac{5}{4}mg$	dM1
	$\omega^2 \le \dfrac{g}{4a}$	A1
	$\omega = \dfrac{1}{2}\sqrt{\dfrac{g}{a}}$ o.e.	A1
		[10]

B1	$\sin\theta = \frac{4}{5}$ or $\cos\theta = \frac{3}{5}$ seen (or reversed if they have angle of string to vertical)
M1	Resolving horizontally, with two tensions (T_a resolved) and acceleration in either form.
A1	Both tensions correct
A1	Correct form of acceleration
M1	Attempt at vertical equation, with T_a resolved and weight not resolved
A1	Correct equation
dM1	Eliminate T_a. Dependent on both previous M marks.
dM1	Set $T_b \le \frac{5}{4}mg$ to form inequality in ω^2. Trig need not be substituted. Dependent on previous M mark. Condone use of $<$
A1	Correct inequality for ω^2 Condone use of $<$
A1	$\omega = \frac{1}{2}\sqrt{\frac{g}{a}}$ o.e. Must have used $\le$ for this mark.

ALT – for last 4 marks

	$8ma\omega^2 = T_b + \dfrac{mg}{\sin\theta} \times \cos\theta$	dM1
	$8ma{\omega_{Max}}^2 = \left(\dfrac{5}{4} + \dfrac{3}{4}\right)mg$	dM1
	${\omega_{Max}}^2 = \dfrac{g}{4a}$	A1
	$\omega = \dfrac{1}{2}\sqrt{\dfrac{g}{a}}$ o.e.	A1

dM1	Eliminate T_a
dM1	Set $T_b = \dfrac{5}{4}mg$ to form and equation in ${\omega_{Max}}^2$ Some justification required that ω_{Max} will be when tension is greatest in PB. Trig need not be substituted.
A1	Correct expression for ${\omega_{Max}}^2$
A1	$\omega = \frac{1}{2}\sqrt{\frac{g}{a}}$ o.e.

5 (a)	Vol = $\pi\int_0^{\frac{\pi}{4}} y^2 \mathrm{d}x = \pi\int_0^{\frac{\pi}{4}} \sin^2 x\,\mathrm{d}x$	M1
	$=\pi\int_0^{\frac{\pi}{4}} \frac{1}{2}(1-\cos 2x)\,\mathrm{d}x$	dM1
	$=\frac{\pi}{2}\left[x-\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}} = \frac{\pi}{2}\left(\frac{\pi}{4}-\frac{1}{2}\right)=\frac{\pi}{8}(\pi-2)$ *	dM1A1 (4)
(b)	$\pi\int_0^{\frac{\pi}{4}} y^2 x\,\mathrm{d}x = \pi\int_0^{\frac{\pi}{4}} x\sin^2 x\,\mathrm{d}x$	M1
	$=\pi\int_0^{\frac{\pi}{4}} \frac{1}{2}x(1-\cos 2x)\,\mathrm{d}x$	dM1
	$=\frac{\pi}{2}\left[\frac{x^2}{2}\right]_0^{\frac{\pi}{4}} - \frac{\pi}{2}\int_0^{\frac{\pi}{4}} x\cos 2x\,\mathrm{d}x$	
	$\frac{\pi^3}{64}, -\frac{\pi}{2}\left[x\times\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}} + \frac{\pi}{2}\int_0^{\frac{\pi}{4}} \frac{1}{2}\sin 2x\,\mathrm{d}x$	B1,dM1
	$=\frac{\pi^3}{64} \quad -\frac{\pi}{2}\times\frac{\pi}{8}\times 1 - \frac{\pi}{2}\left[\frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{4}}$	dM1
	$=\frac{\pi^3}{64}-\frac{\pi^2}{16}-\frac{\pi}{8}(0-1)=\frac{\pi^3}{64}-\frac{\pi^2}{16}+\frac{\pi}{8} \quad (=0.2603...)$	A1
	$\bar{x}=\left(\frac{\pi^3}{64}-\frac{\pi^2}{16}+\frac{\pi}{8}\right)\div\frac{\pi}{8}(\pi-2) \quad =0.5806...=0.581$	M1A1 (8)
(c)	$\tan\theta = \dfrac{\frac{\pi}{4}-\bar{x}}{\sin\frac{\pi}{4}}$	M1
	$\tan\alpha = \dfrac{\frac{\pi}{4}}{\sin\frac{\pi}{4}}$	B1
	reqd angle $=\alpha-\theta=48.00...-16.12...=31.8...=32°$	M1A1 (4) [16]

(a)	
M1	Set up the volume integral in terms of x. π and limits not needed (ignore any shown).
dM1	Attempt to use double angles to prepare integral. Allow $\frac{1}{2}(\pm 1 \pm \cos 2x)$
dM1	Attempt integration and substitute of correct limits. Depends on previous M mark.
A1	Arrive at given result, with no errors seen. If π only appears at end, there must be some justification.
(b)	**Note:** If correct algebraic integration seen, but no evidence of use of limits, a correct final answer implies the B1 and A marks.
M1	Attempt $\int xy^2\,\mathrm{d}x$ in terms of x, π and limits not needed (ignore any shown)
dM1	Use double angle identity and split integral into two parts, ready for integration. Dependent on first M1.
B1	$\frac{\pi^3}{64}$ or $\frac{\pi^2}{64}$ if π cancelled
dM1	Attempt to start Integration by Parts. Dependent on previous M1.
dM1	Complete integration and substitute limits. Dependent on previous M1.
A1	Correct result (exact or decimal). Accept $\frac{\pi^2}{64}-\frac{\pi}{16}+\frac{1}{8}$
M1	Use $\bar{x}=\dfrac{\int xy^2\mathrm{d}x}{\int y^2\mathrm{d}x}$ π and ρ must be in both or neither.
A1cao	0.581 **must be 3s.f.**
Alt	Integral from (a) can be used, rather than splitting the integral
	$\int \frac{1}{2}x(1-\cos 2x)\,\mathrm{d}x=\frac{1}{2}x\left(x-\frac{\sin 2x}{2}\right)-\frac{1}{2}\int\left(x-\frac{\sin 2x}{2}\right)dx$
	For second M1 we just need the use of the double angle identity and clear evidence of the use of the result from (a).
M1	Use their $\bar{x}$ correctly to find $\tan\theta$
B1	Correct expression for $\tan\alpha$ seen.
M1	Correct strategy (based on their $\bar{x}$) to find the required angle.
A1	32° cao

6(a)	At B: $mg\cos 60° = \dfrac{mv^2}{r}$	M1A1A1
	$v = \sqrt{\dfrac{rg}{2}}$ *	A1 (4)
(b)	$\dfrac{1}{2}m\left(u^2 - v^2\right) = 2mgr\sin 30°$	M1A1A1
	$u^2 = 4gr \times \dfrac{1}{2} + \dfrac{rg}{2}$	
	$u = \sqrt{\dfrac{5rg}{2}}$	A1 (4)
(c)	$0^{(2)} = \left(v\sin 60\right)^2 - 2gh$	M1A1
	$h = \dfrac{3v^2}{8g} = \dfrac{3r}{16}$	A1
	Height above $C = \dfrac{3r}{16} + r + r\cos 60°$	M1
	$= \dfrac{27}{16}r$	A1 (5)
		[13]

(a)	
M1	Attempt at equation of motion along the radius, with weight resolved and acceleration in either form. If R is included, it must be set to zero before this mark is awarded.
A1A1	Fully correct equation. -1 for each error. Acceleration must be in correct form.
A1*	Given answer reached from fully correct working. There must be at least one line of working between At B: $mg\cos 60° = \frac{mv^2}{r}$ and the given result.
(b)	
M1	Energy equation with 2 KE terms and a change in GPE. Use of SUVAT scores M0. The work might be seen in (a), but must be used in (b).
A1	Correct GPE or change in KE.
A1	Fully correct equation.
A1	Correct expression for u.
(c)	
M1	Complete SUVAT method to find height gained after leaving sphere. Initial velocity must be resolved. If energy used, they must take account of the fact that there is still KE at the highest point.
A1	Fully correct equation.
A1	Correct height gained.
M1	Complete method to find height above C, by adding 3 distances. Independent of previous M mark, as long a dimensionally correct height gained has been found.
A1	$\frac{27}{16}r\left(=1.6875r\right)$ **cso**

7 (a)	$T=\dfrac{\lambda \frac{2}{5}l}{l}$	
	$mg\sin\alpha=\dfrac{3}{5}mg=\dfrac{2}{5}\lambda$	M1A1
	$\lambda=\dfrac{3}{2}mg$ *	A1 (3)
(b)(i)	$\dfrac{3}{5}mg-T=m\ddot{x}$	M1
	$\dfrac{3}{5}mg-\dfrac{1}{l}\left(\dfrac{2}{5}l+x\right)\times\dfrac{3}{2}mg=m\ddot{x}$	dM1A1
	$\ddot{x}=-\dfrac{3g}{2l}x \quad \therefore$ SHM	A1
(ii)	B is the equilibrium position or $x=0$ at B	B1 (5)
(c)	Time $=\dfrac{1}{4}\times\dfrac{2\pi}{\omega}=\dfrac{\pi}{2}\sqrt{\dfrac{2l}{3g}}$ oe	M1A1ft (2)
(d)	$-\dfrac{2}{5}l=\dfrac{4}{5}l\cos\omega t=\dfrac{4}{5}l\cos\sqrt{\dfrac{3g}{2l}}t$	M1
	$\dfrac{2}{3}\pi=\sqrt{\dfrac{3g}{2l}}t\Rightarrow t=\dfrac{2}{3}\pi\sqrt{\dfrac{2l}{3g}}$ oe	A1ft
	At nat length $v^2=\omega^2\left(a^2-x^2\right)=\dfrac{3g}{2l}\left(\dfrac{16}{25}l^2-\dfrac{4}{25}l^2\right)=\dfrac{36gl}{50}$	M1A1ft
	Once string is slack accel is $\dfrac{3}{5}g$ down the plane.	
	$0=\dfrac{6}{5}\sqrt{\dfrac{gl}{2}}-\dfrac{3}{5}gt'$	M1
	Total time $=\dfrac{2}{3}\pi\sqrt{\dfrac{2l}{3g}}+2\sqrt{\dfrac{l}{2g}}$ oe	A1 **cso** (6)
		[16]

(a)	
M1	Resolve down the slope. Weight must be resolved and Hooke's Law attempted.
A1	Correct equation, including correct use of (unsimplified) Hooke's Law. Trig need not be substituted.
A1*	Given result obtained convincingly. We must see $\frac{2l}{5}$ and trig must be substituted before final result given.
(b) (i)	
M1	Equation of motion along slope, in T. Allow with *ma*.
dM1	Hooke's Law applied, with a variable extension. Allow with *ma*. Dependent on previous M mark.
A1	Correct unsimplified equation. Must now be $m\ddot{x}$
A1	$\ddot{x} = -\frac{3g}{2l}x \quad \therefore$ SHM Must have conclusion.
(ii)	
B1	*B* is the equilibrium position **or** $x = 0$ at *B*.
(c)	
M1	Complete method to find a quarter period for their ω. Must have come from an equation of the form $\ddot{x}$ or $a = \pm\omega^2 x$, but ignore dimensions of their ω.
A1ft	Correct time (ft their ω)
(d)	
M1	Complete method to find the time to string becoming slack. Must be using $x = \pm\frac{2}{5}l, a = \frac{4}{5}l$
	If using sin $\frac{2}{5}l = \frac{4}{5}l\sin\omega t_1 \rightarrow t_1 = \frac{\pi}{6}\sqrt{\frac{2l}{3g}} \quad \therefore t = \frac{\pi}{2}\sqrt{\frac{2l}{3g}} + \frac{\pi}{6}\sqrt{\frac{2l}{3g}}$
A1ft	Correct expression for time until string becomes slack. Follow through their ω.
M1	Complete method to find speed when string becomes slack
	Energy alternative $\frac{1}{2}mv^2 = \frac{1}{2}\times\frac{3mg}{2}\times\left(\frac{6l}{5}\right)^2 - mg\times\frac{6l}{5}\times\frac{3}{5}$
A1ft	Correct v or v^2 Follow through their ω.
M1	Complete method to find **total** time for *P* to come to rest.
A1 **cso**	Correct time, in any form. **Must have come from correct working in whole question.**

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