# Mark Scheme (Results)

Summer 2019

Pearson Edexcel International GCE
In IAL Core Mathematics C34 (WMA02/01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL IAL MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 125

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

**Method mark for solving 3 term quadratic:**

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \quad \text{leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \quad \text{leading to } x = \dots$$

2. Formula

Attempt to use the correct formula (with values for $a$, $b$ and $c$).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0,$ leading to $x = \dots$

**Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

**Answers without working**

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

| Qu Number | Scheme | | Marks |
|---|---|---|---|
| **1(a)** | $2x^3 = \pm x \pm 20$ or $x^3 = \dfrac{\pm x \pm 20}{2}$ $\Rightarrow x = \sqrt[3]{\dfrac{\pm x \pm 20}{2}}$ | Correct order of operations including cube root. The " = 0" does not have to be seen initially and can be implied by e.g. $2x^3 = \pm x \pm 20$. | M1 |
| | $x = \sqrt[3]{10 - \dfrac{1}{2}x}$ or $x = \sqrt[3]{\left(10 - \dfrac{1}{2}x\right)}$ | Correct equation or exact equivalent e.g. $x = \sqrt[3]{10 - 0.5x}$ or $x = \sqrt[3]{-0.5x + 10}$ with no errors or incorrect statements. The vinculum should encompass both terms and as a rule of thumb should at least go beyond the "–" or the "+". $x = \pm\sqrt[3]{\ldots}$ scores A0. Isw once the correct answer is obtained. | A1 |
| | | | **(2)** |
| **(a) Way 2** | $x = \sqrt[3]{a - bx} \Rightarrow x^3 = a - bx$ $\Rightarrow x^3 + bx - a = 0$ $\Rightarrow 2x^3 + 2bx - 2a = 0$ $\Rightarrow a = \ldots, b = \ldots$ | Correct order of operations e.g. cubes, collects to one side and multiplies by 2. Then compares coefficients to establish values for *a* and *b*. | M1 |
| | $\Rightarrow a = 10,\ b = \dfrac{1}{2}$ | Correct values and apply isw if necessary. | A1 |
| **(b)** | $x_2 = \sqrt[3]{10 - \dfrac{1}{2} \times 2.1}$ | Substitutes $x_1 = 2.1$ into $x_{n+1} = \sqrt[3]{a - bx_n}$ with their numerical values of *a* and *b* in order to find $x_2$. Can be implied by awrt 2.076 if *a* and *b* are correct otherwise may need to check. | M1 |
| | $(x_2 =)$ awrt 2.076 $(x_3 =)$ awrt 2.077 | Correct values. | A1 |
| | | | **(2)** |
| **(c)** | $\mathrm{f}(2.0765) = -0.016\ldots$ $\mathrm{f}(2.0775) = 0.011\ldots$ | Chooses a suitable interval for *x*, which is within $2.077 \pm 0.0005$ and attempts to evaluate $\mathrm{f}(x) = 2x^3 + x - 20$ for both values and obtains at least one value correct to 1 sig fig (rounded or truncated). | M1 |
| | Sign change (negative, positive) therefore root. | Both values correct awrt (or truncated) 1 sf, sign change (or e.g. < 0, > 0 or f(2.0765).f(2.0775) < 0 or f(2.0765) < 0 < f(2.0775)) and a minimal conclusion e.g. therefore root. Allow tick, QED, hash, square box, smiley face etc. | A1 |
| | **Attempts at repeated iteration scores no marks in (c)** | | |
| | | | **(2)** |
| **(d)** | 0.077 | Cao | B1 |
| | | | **(1)** |
| | | | **[7 marks]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | Note that use of ln $kx$ for ln $x$ is acceptable throughout. | | |
| **2(a)** | $\int \frac{4x+3}{x}\,dx \to \int ...+\frac{b}{x}\,dx = ..+..\ln x$<br>Attempts to divide to obtain $...+\frac{b}{x}$ and uses $\int \frac{1}{x}\,dx = \ln x$ or $\int \frac{1}{x}\,dx = \ln kx$ | | M1 |
| | $= 4x + 3\ln x + (c)$ | There is no requirement for the + $c$ | A1 |
| | | | **(2)** |
| **(a) Way 2** | $\int \frac{4x+3}{x}\,dx = \int (4x+3)x^{-1}\,dx = (4x+3)\ln x - \int 4\ln x\,dx$<br>$\int \frac{4x+3}{x}\,dx = (4x+3)\ln x - \int ...\ln x\,dx = (4x+3)\ln x - 4x\ln x + kx$<br>This method requires 2 applications of parts to obtain an expression of this form | | M1 |
| | $=(4x+3)\ln x - 4x\ln x + 4x(+c)$ | There is no requirement for the + $c$ | A1 |
| **(a) Way 3** | $\int \frac{4x+3}{x}\,dx = \int (4x+3)x^{-1}\,dx = (2x^2+3x)x^{-1} + \int (2x^2+3x)x^{-2}\,dx$<br>$=(2x+3)+\int (2+3x^{-1})\,dx = 2x+3+2x+3\ln x(+c)$<br>$\int \frac{4x+3}{x}\,dx = (2x^2+3x)x^{-1} + ... + ...\ln x$<br>This method requires the applications of parts to obtain an expression of this form | | M1 |
| | $=(2x^2+3x)x^{-1}+2x+3\ln x(+c)$ | There is no requirement for the + $c$ | A1 |

| | | | |
|---|---|---|---|
| **2(b)** | $\frac{dy}{dx} = \frac{(4x+3)y^{\frac{1}{2}}}{x} \Rightarrow \int \frac{1}{y^{\frac{1}{2}}}dy = \int \frac{(4x+3)}{x}dx$<br>Separates the variables correctly.<br>Accept $\int \frac{1}{y^{\frac{1}{2}}}dy = \int \frac{(4x+3)}{x}dx$ or equivalent.<br>With or without the integral signs and possibly without the "d$x$" and/or "d$y$"<br>so look for $\frac{1}{y^{\frac{1}{2}}} = \frac{(4x+3)}{x}$ | | B1 |
| | $2y^{\frac{1}{2}} = 4x + 3\ln x + c$ | Look for $ky^{\frac{1}{2}} =$ their ($a$)<br>or $ky^{\frac{1}{2}} =$ an attempt at $\int \frac{4x+3}{x}\,dx$ | M1 |
| | | $2y^{\frac{1}{2}} = 4x + 3\ln x + c$ or equivalent including the $+c$ | A1 |
| | $x = 1, y = 25$<br>$\Rightarrow 2(25)^{\frac{1}{2}} = 4(1) + 3\ln(1) + c \Rightarrow c = \ldots$ | Substitutes $x$ = 1 and $y$ = 25 into their integrated equation and proceeds to obtain a value for $c$. | M1 |
| | $y = \left(2x + \frac{3}{2}\ln x + 3\right)^2$ | Correct equation including "$y$ =". The $2x + \frac{3}{2}\ln x + 3$ can be in any equivalent correct form. | A1 |
| | | | **(5)** |
| | | | **[7 marks]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **3(a)** | $k=3$ | Correct value | B1 |
| | | | **(1)** |
| **(b)** | $\sec^2\theta = 1+\tan^2\theta \Rightarrow y = 1+\left(\dfrac{x}{\sqrt{3}}\right)^2$ | Attempts to use $1+\tan^2\theta=\sec^2\theta$ with the given parametric equations to obtain an equation in terms of *x* and *y* only. | M1 |
| | $\Rightarrow y = 1+\dfrac{1}{3}x^2$ | $y=1+\dfrac{1}{3}x^2$ or $\mathrm{f}(x)=1+\dfrac{1}{3}x^2$ <br> (Allow $y/\mathrm{f}(x)=\dfrac{3+x^2}{3}$) but not <br> $y=1+\left(\dfrac{x}{\sqrt{3}}\right)^2$ | A1 |
| | Note that the following is also valid: <br> $\tan\theta=\dfrac{x}{\sqrt{3}} \Rightarrow \cos\theta=\dfrac{\sqrt{3}}{\sqrt{x^2+3}} \Rightarrow \cos^2\theta=\dfrac{3}{x^2+3}$ <br> $y=\sec^2\theta \Rightarrow \dfrac{1}{\cos^2\theta}=\dfrac{x^2+3}{3}$ <br> M1: For using correct trigonometry and $\sec\theta=\dfrac{1}{\cos\theta}$ to obtain an equation in terms of *x* and *y* only. <br> A1: As above | | |
| | | | **(2)** |
| **(c)** | $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2}{3}x$ | Differentiates their f(*x*) with evidence of $x^n \to x^{n-1}$ or for differentiating to a correct form for their function. | M1 |
| | $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=1}=\dfrac{2}{3}(1)=\dots$ | Attempts to find their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at *x* = 1 (or their attempt at *x*) | M1 |
| | (Gradient =) $\dfrac{2}{3}$ | For $\dfrac{2}{3}$ | A1 |
| | | | **(3)** |
| **(c) Way 2** | $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=\sqrt{3}\sec^2\theta,\ \dfrac{\mathrm{d}y}{\mathrm{d}\theta}=2\sec^2\theta\tan\theta$ <br> $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2\sec^2\theta\tan\theta}{\sqrt{3}\sec^2\theta}$ | Attempts $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y/\mathrm{d}\theta}{\mathrm{d}x/\mathrm{d}\theta}=\dfrac{..\sec^2\theta\tan\theta}{..\sec^2\theta}$ | M1 |
| | $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2\sec^2\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right)}{\sqrt{3}\sec^2\left(\frac{\pi}{6}\right)}$ | Attempts to find their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at $\theta=\dfrac{\pi}{6}$ | M1 |
| | (Gradient =) $\dfrac{2}{3}$ | For $\dfrac{2}{3}$ | A1 |
| | | | **[6 marks]** |

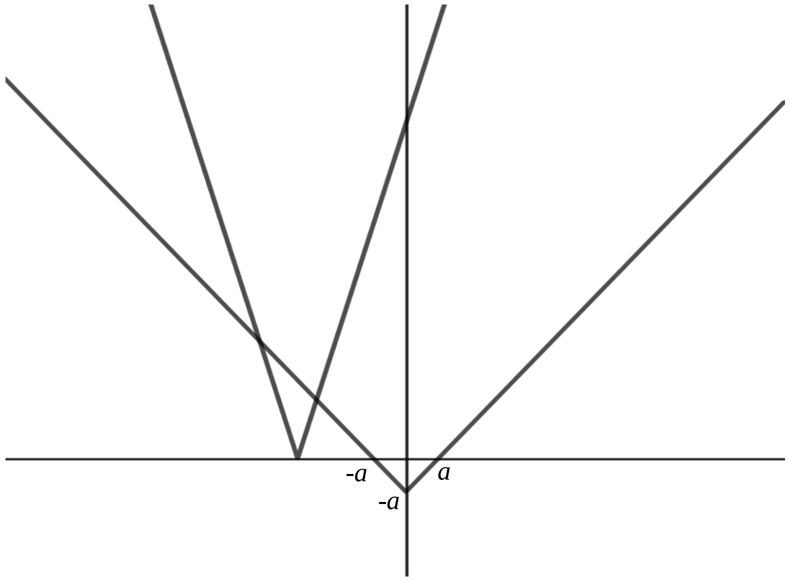
| Question Number | Scheme | | Marks |
|---|---|---|---|
| | **Mark (a) and (b) together** | | |
| **4(a)** | Accept alternative notation for $\frac{\mathrm{d}y}{\mathrm{d}x}$ throughout e.g. $y'$<br>You can ignore a spurious "$\frac{\mathrm{d}y}{\mathrm{d}x} =$" for the first 3 marks | | |
| | $3y\mathrm{e}^{-2x} = 4x^2 + y^2 + 2 \Rightarrow \overline{\overline{3\mathrm{e}^{-2x}\frac{\mathrm{d}y}{\mathrm{d}x} - 6y\mathrm{e}^{-2x}}} = 8x + \underline{2y\frac{\mathrm{d}y}{\mathrm{d}x}}$<br>$\overline{\overline{\text{M1}}}$: Attempts the product rule on $3y\mathrm{e}^{-2x}$ to give $p\mathrm{e}^{-2x}\frac{\mathrm{d}y}{\mathrm{d}x} \pm qy\mathrm{e}^{-2x}$<br>(If the product rule is quoted it must be correct with the +)<br>$\underline{\text{M1}}$: Attempts the chain rule on $y^2$ to give $Ay\frac{\mathrm{d}y}{\mathrm{d}x}$<br>A1: For correct differentiation: $3\mathrm{e}^{-2x}\frac{\mathrm{d}y}{\mathrm{d}x} - 6y\mathrm{e}^{-2x} = 8x + 2y\frac{\mathrm{d}y}{\mathrm{d}x}$<br>May not be seen as an equation so allow A1 if $3\mathrm{e}^{-2x}\frac{\mathrm{d}y}{\mathrm{d}x} - 6y\mathrm{e}^{-2x}$ and $8x + 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ are seen separately | | $\overline{\overline{\text{M1}}}$ $\underline{\text{M1}}$<br>A1 |
| | Note that this notation is acceptable:<br>$3y\mathrm{e}^{-2x} = 4x^2 + y^2 + 2 \Rightarrow \overline{\overline{3\mathrm{e}^{-2x}\mathrm{d}y - 6y\mathrm{e}^{-2x}\mathrm{d}x}} = 8x\mathrm{d}x + \underline{2y\,\mathrm{d}y}$<br>$\overline{\overline{\text{M1}}}$: Attempts the product rule on $3y\mathrm{e}^{-2x}$ to give $p\mathrm{e}^{-2x}\mathrm{d}y \pm qy\mathrm{e}^{-2x}\mathrm{d}x$<br>$\underline{\text{M1}}$: Attempts the chain rule on $y^2$ to give $Ay\,\mathrm{d}y$<br>A1: For correct differentiation: $3\mathrm{e}^{-2x}\mathrm{d}y - 6y\mathrm{e}^{-2x}\mathrm{d}x = 8x\,\mathrm{d}x + 2y\,\mathrm{d}y$<br>Then as below but collects terms in $\mathrm{d}y$ (must be two) and makes $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject of the formula | | |
| | $\left(3\mathrm{e}^{-2x} - 2y\right)\frac{\mathrm{d}y}{\mathrm{d}x} = 8x + 6y\mathrm{e}^{-2x}$<br>$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$ | Collects terms in $\frac{\mathrm{d}y}{\mathrm{d}x}$ (must be two – one from the product and one from the $\underline{2y\frac{\mathrm{d}y}{\mathrm{d}x}}$) and makes $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject of the formula | M1 |
| | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8x + 6y\mathrm{e}^{-2x}}{3\mathrm{e}^{-2x} - 2y}$ | Correct expression (allow equivalent correct forms) | A1 |
| | | | **(5)** |

| | | | |
|---|---|---|---|
| **4(b)** | $x=0, y=2 \Rightarrow$<br>$\frac{dy}{dx}=\frac{8(0)+6(2)e^{-2(0)}}{3e^{-2(0)}-2(2)}=(-12)$ | Substitutes $x=0, y=2$ into their $\frac{dy}{dx}$ or into their differentiated equation and makes $\frac{dy}{dx}$ the subject. May be implied by their value for $\frac{dy}{dx}$. | M1 |
| | $y-2=-\frac{1}{"-12"}(x-0)$ | Uses correct form of the equation of the normal. Look for<br>$y-2=-\frac{1}{\textit{their } dy/dx\|_{(0,2)}}(x-0)$<br>where their d$y$/d$x$ is non-zero or not undefined.<br>**Dependent on the first method mark.** | dM1 |
| | $y=\frac{1}{12}x+2$ | Cao cso | A1 |
| | | | **(3)** |
| | | | **[8 marks]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **5(a)** | $t=0, \theta=38 \Rightarrow 38=20+Ae^{-k\times 0}$ | For substituting $t=0$ and $\theta=38$ into $\theta=20+Ae^{-kt}$ | M1 |
| | $\Rightarrow A=18$ | Correct value for $A$ | A1 |
| | ***A* = 18 with no working scores both marks** | | |
| | | | **(2)** |
| **(b)** | $t=16, \theta=24.5 \Rightarrow 24.5=20+"18"e^{-k\times 16}$ | For substituting $t=16$ and $\theta=24.5$ into $\theta=20+$ their $"A"e^{-kt}$ | M1 |
| | $\Rightarrow 18e^{-k\times 16}=4.5$ or $e^{-k\times 16}=\frac{1}{4}$ | This mark is for a correct equation with the **constants combined.** Allow equivalent correct equations e.g. $e^{16k}=4$ | A1 |
| | $\Rightarrow e^{16k}=4 \Rightarrow 16k=\ln 4$<br>or<br>$\Rightarrow \ln 18e^{-k\times 16}=\ln 4.5 \Rightarrow \ln 18+\ln e^{-k\times 16}=\ln 4.5 \Rightarrow \ln e^{-k\times 16}=\ln\frac{1}{4}$<br>$\Rightarrow -16k=\ln\frac{1}{4}$<br>Uses correct log or exponential work to move from:<br>$e^{\pm nk}=C$ to $\pm nk=\alpha\ln C$ **or** $pe^{\pm nk}=q$ to $\pm nk=\alpha\ln\beta$ | | M1 |
| | $-16k=\ln\frac{1}{4} \Rightarrow k=-\frac{1}{16}\ln\frac{1}{4}=\frac{1}{8}\ln 2*$<br>Shows that $k=\frac{1}{8}\ln 2$<br>There must be **at least one intermediate line** between their $\pm nk=\alpha\ln C$ or their $\pm nk=\alpha\ln\beta$ and the printed answer.<br>So for example $-16k=\ln\frac{1}{4} \Rightarrow k=\frac{1}{8}\ln 2*$ scores A0 as there is no intermediate line. | | A1* |
| | Note:<br>The marks in part (b) can be scored by using $\theta=20+Ae^{-kt}$ and substituting 2 out of: $A$ = 18, $\theta=24.5$, $k=\frac{1}{8}\ln 2$ to show that the 3rd variable is correct followed by a conclusion e.g. so $k=\frac{1}{8}\ln 2$ | | |
| | | | **(4)** |
| **(c)** | $t=40 \Rightarrow \theta=20+"18"e^{-\frac{1}{8}\ln 2\times 40}$ | Substitutes $t$ = 40 into the given equation with their $A$ and the given value of $k$ to obtain a value for $\theta$ | M1 |
| | $\Rightarrow \theta=$ awrt $20.6(°C)$ | Awrt 20.6 | A1 |
| | **Correct answer only scores both marks** | | |
| | | | **(2)** |

| | | |
|---|---|---|
| **5(d)** | **Examples:**<br>• The lower limit is 20<br>• $\theta > 20$<br>• As $t$ tends to infinity temperature tends to 20<br>• The temperature cannot go below 20<br>• $e^{-kt}$ tends towards zero so the temperature tends to 20<br>• $e^{-kt}$ is always positive so the temperature is always bigger than 20<br>• Substitutes $\theta = 19$ in $\theta = 20 + "18"e^{-kt}$ (may be implied by e.g. $e^{-kt} = -\frac{1}{18}$) and states e.g. that you cannot find the log of a negative number or "which is not possible"<br><br>Do not accept $e^{-kt}$ cannot be negative without reference to the "20" | B1 |
| | | **(1)** |
| | | **[9 marks]** |

| Question Number | Scheme | Marks |
|---|---|---|
| **6 (a)(i)(ii)** | **Mark (a)(i) and (ii) together** | |
| | $(0,10a)$ or $\left(-\frac{5}{2}a,\ 0\right)$ <br> **or** <br> $(x=0, y=10a)$ or $\left(y=0, x=-\frac{5}{2}a\right)$ <br> One correct coordinate pair. Allow as separate coordinates or clear sight of the "0's" and allow $\lvert 10a \rvert$ for $10a$ and allow equivalents for $-\frac{5}{2}a$ e.g. $-\frac{10}{4}a$. <br> **Ignore labelling of parts and points** | B1 |
| | $(0,10a)$ **and** $\left(-\frac{5}{2}a,\ 0\right)$ <br> **or** <br> $(x=0, y=10a)$ or $\left(y=0, x=-\frac{5}{2}a\right)$ <br> Two correct coordinate pairs. Allow as separate coordinates or clear sight of the "0's" and allow $\lvert 10a \rvert$ for $10a$ and allow equivalents for $-\frac{5}{2}a$ e.g. $-\frac{10}{4}a$. <br> **Ignore labelling of parts and points** | B1 |
| | You can condone missing brackets e.g. $-\frac{5}{2}a,\ 0$ or $0,\ 10a$ but if the "0's" are not evident in **either** case, e.g. if all that is seen is 10a and $-\frac{5}{2}a$ score B1B0 <br> If the coordinates are consistently the wrong way round <br> e.g. $(10a, 0)$ and $\left(0,\ -\frac{5}{2}a\right)$ score **B1B0** <br> If the coordinates are on the sketch, the zero's have to be seen to score both marks but score **B1B0** if the $10a$ and $-\frac{5}{2}a$ are seen in the correct places | |
| | | **(2)** |

| | | | |
|---|---|---|---|
| **6(b)** | <br>V (or tick) shape with vertex on the negative $y$-axis and branches pointing upwards with one branch to the left and one branch to the right of the $y$-axis with part of the V in all 4 quadrants – the left branch does not necessarily need to cross the other "V"<br>(Ignore gradient as long as it is a V shape)<br>Do not be overly concerned by lack of symmetry.<br>Allow the diagram above the question to be adapted or a separate sketch. | | B1<br>**M1 on ePEN** |
| | Intersections at $(-a,0),(a,0)$ and $(0,-a)$ only.<br>Can be seen as coordinates or as shown in the diagram.<br>If the coordinates are shown away from the sketch they must appear as $(-a,0),(a,0)$ and $(0,-a)$ and must correspond with the sketch.<br>If there is any ambiguity the sketch has precedence. | | B1<br>**A1 on ePEN** |
| | | | **(2)** |
| **6(c)** | $-x-a=4x+10a \Rightarrow x=...$<br>**or**<br>$-x-a=-4x-10a \Rightarrow x=...$ | Attempts to solve $-x-a=4x+10a$ or $-x-a=-4x-10a$ or equivalent equations **to obtain *x* in terms of *a*.** | M1 |
| | $x=-\frac{11}{5}a$ **or** $-3a$ | One correct. Allow $-\frac{9}{3}a$ for $-3a$ | A1 |
| | $x=-\frac{11}{5}a$ **and** $-3a$ | Both correct and no other values.<br>Allow $-\frac{9}{3}a$ for $-3a$. | A1 |
| | **Note that attempts to square both sides and to solve the resulting quadratic generally scores no marks. However if you think such attempts deserve credit then use Review.** | | |
| | | | **(3)** |
| | | | **[7 marks]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **7(a)** | $5\cos\theta - 3\sin\theta = R\cos(\theta + \alpha)$ | | |
| | $R = \sqrt{5^2 + 3^2} = \sqrt{34}$ | $R = \sqrt{34}$ $\left(R = \pm\sqrt{34} \text{ is B0}\right)$ | B1 |
| | $\tan\alpha = \pm\frac{3}{5}$, $\tan\alpha = \pm\frac{5}{3}$ $\Rightarrow$ $\alpha = ...$<br>(Also allow $\cos\alpha = \pm\frac{5}{\sqrt{34}}$ or $\pm\frac{3}{\sqrt{34}}$, $\sin\alpha = \pm\frac{3}{\sqrt{34}}$ or $\pm\frac{5}{\sqrt{34}}$ $\Rightarrow$ $\alpha = ...$, where "$\sqrt{34}$" is their $R$.) | | M1 |
| | $\alpha = \arctan\left(\frac{3}{5}\right) = \text{awrt } 0.5404$ | Anything that rounds to 0.5404<br>(Degrees is 30.96… and scores A0) | A1 |
| | | | **(3)** |

| | | |
|---|---|---|
| **7(b)** | $6+2.5\cos\left(\frac{4\pi t}{25}\right)-1.5\sin\left(\frac{4\pi t}{25}\right)=4.6\Rightarrow\frac{\sqrt{34}}{2}\cos\left(\frac{4\pi t}{25}+0.5404\right)=4.6-6$<br>$\Rightarrow\cos\left(\frac{4\pi t}{25}+"0.5404"\right)=...$<br>Uses part (a) and proceeds as far as<br>$\cos\left(\frac{4\pi t}{25}\pm\text{their }0.5404\right)=k$ or $\cos\ \theta\pm\text{their }0.5404\ =k$ or<br>$\cos\ t\pm\text{their }0.5404\ =k$ where $\lvert k\rvert<1$ . | M1 |
| | $\cos\left(\frac{4\pi t}{25}+"0.5404"\right)=-0.48$ \| Allow:<br>$\cos\left(\frac{4\pi t}{25}\pm\text{their }0.5404\right)=\text{awrt }-0.48$<br>**or** $\cos\ \theta\pm\text{their }0.5404\ =\text{awrt }-0.48$<br>**or** $\cos\ t\pm\text{their }0.5404\ =\text{awrt }-0.48$<br>May see $-\frac{7\sqrt{34}}{85}$ or $-\frac{2.8}{\sqrt{34}}$ for – 0.48 | A1 |
| | $\frac{4\pi t}{25}+"0.5404"=2.07\Rightarrow t=...$ **or**<br>$\frac{4\pi t}{25}+"0.5404"=2\pi-2.07\ =4.21\ \Rightarrow t=...$<br>NB 2.07… may be seen as π – 1.07 and 4.21… may be seen as π + 1.07<br>$\cos\left(\frac{4\pi t}{25}\pm\text{their }0.5404\right)=k\Rightarrow t=..$ by first taking invcos then adds or subtracts their 0.5404 and applies $\frac{4\pi t}{25}$ to obtain a value for $t$.<br>**Dependent on the previous method mark and may be implied by obtaining a value for *t* of awrt 3 or awrt 7.** | **d**M1 |
| | awrt 3.05 or awrt 7.3 \| Allow awrt 3.05 **or** awrt 7.3 | A1 |
| | $\frac{4\pi t}{25}\pm"0.5404"=2\pi-2.07\Rightarrow t=...$ **and** $\frac{4\pi t}{25}\pm"0.5404"=2.07\Rightarrow t=...$<br>For a correct method to find a different value of $t$ in the range<br>**Dependent on both previous method marks.** | **dd**M1 |
| | 3:03 or 15:03 or 3hrs 3min or 183 minutes<br>**and**<br>7:18 or 19:18 or 7hrs 18min or 438 minutes | A1 |
| | | **(6)** |
| | | **[9 marks]** |

<table>
<tr><th>Qu Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="3">8(a)</td><td colspan="2">$$\mathrm{f}(x)=\frac{6x+2}{3x^2+5}\Rightarrow \mathrm{f}'(x)=\frac{6\left(3x^2+5\right)-6x\left(6x+2\right)}{\left(3x^2+5\right)^2}$$<br>or<br>$$\mathrm{f}(x)=\left(6x+2\right)\left(3x^2+5\right)^{-1}\Rightarrow \mathrm{f}'(x)=6\left(3x^2+5\right)^{-1}-6x\left(6x+2\right)\left(3x^2+5\right)^{-2}$$<br>M1 for $\dfrac{\alpha\left(3x^2+5\right)-\beta x\left(6x+2\right)}{\left(3x^2+5\right)^2}$ or $\alpha\left(3x^2+5\right)^{-1}-\beta x\left(6x+2\right)\left(3x^2+5\right)^{-2}$<br>Condone obvious slips and bracketing errors e.g.<br>$\dfrac{(6)3x^2+5-(6x)6x+2}{\left(3x^2+5\right)^2}$ as long as the intention is clear i.e. recovered in subsequent working<br>If the product or quotient rule is quoted, it must be correct<br>A1: Fully correct derivative in any form</td><td>M1 A1</td></tr>
<tr><td colspan="2">$$\Rightarrow \mathrm{f}'(x)=\frac{30-12x-18x^2}{\left(3x^2+5\right)^2}$$<br>Correct expression or equivalent e.g. $\mathrm{f}'(x)=\dfrac{-6\left(3x^2+2x-5\right)}{\left(3x^2+5\right)^2}$<br>e.g. $\mathrm{f}'(x)=\dfrac{-6\left(3x^2+2x-5\right)}{9x^4+30x^2+25}$<br>Apply isw and award this mark once a correct expression is seen</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>(3)</td></tr>
<tr><td>8(b)</td><td colspan="2">Allow full recovery in (b) following ± (a correct numerator) in (a) or following ± (a correct numerator) and an incorrect denominator in (a)</td><td></td></tr>
<tr><td rowspan="5"></td><td colspan="2">$\mathrm{f}'(x)=0\Rightarrow 30-12x-18x^2=0\Rightarrow -6(3x+5)(x-1)=0\Rightarrow x=...$<br>Sets their numerator = 0 and attempts to solve 2TQ or 3TQ = 0</td><td>M1</td></tr>
<tr><td>$x=-\dfrac{5}{3},\ 1$</td><td>Correct values</td><td>A1</td></tr>
<tr><td>$x=-\dfrac{5}{3}\Rightarrow y=\dfrac{6\left(-\frac{5}{3}\right)+2}{3\left(-\frac{5}{3}\right)^2+5}$<br>or<br>$x=1\Rightarrow y=\dfrac{6(1)+2}{3(1)^2+5}$</td><td>Finds the $y$ coordinate of the turning point from the $x$ coordinate for one of their values. Dependent on the previous method mark.</td><td>dM1</td></tr>
<tr><td colspan="2">$$\Rightarrow\left(-\frac{5}{3},-\frac{3}{5}\right),\ (1,1)$$<br>Correct coordinates but allow $x=\ldots$, $y=\ldots$ and allow equivalent exact fractions/decimals for $-\dfrac{5}{3}$ and/or $-\dfrac{3}{5}$</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>(4)</td></tr>
</table>

| | | | |
|---|---|---|---|
| **8(c)** | Either $\left(\frac{1}{2}\times\text{their}1, \quad\right)$ or $\left(\quad, \text{their}1+4\right)$ | One correct or correct follow through coordinate and allow $x = \ldots, y = \ldots$ | B1ft |
| | $\left(\frac{1}{2}\times\text{their}1,\ \text{their}1+4\right)$ | Both correct or correct follow through coordinates (allow $x = \ldots, y = \ldots$) **<u>but there should be no other points that have clearly not been discarded unless their point is clearly indicated as being the maximum.</u>** | B1ft |
| | | | **(2)** |
| **8(d)** | $-\frac{3}{5}\leqslant y\leqslant\frac{2}{5}$<br>M1: For either end of the inequality including the $\leqslant$ or $\geqslant$ but allow < and/or > for this mark or e.g. max = $\frac{2}{5}$ , min = $-\frac{3}{5}$ but not just values<br>Accept $\frac{2}{5}$ (or equivalent) or follow through on their $-\frac{3}{5}$ (or equivalent) | | M1 |
| | A1: Both ends fully correct with $\leqslant$ and $\geqslant$ but follow through on their $-\frac{3}{5}$ and allow alternative notation such as<br>$\left[-\frac{3}{5}, \frac{2}{5}\right], -\frac{3}{5}\leqslant\text{Range}\leqslant\frac{2}{5}, \left\{y : y\geqslant-\frac{3}{5}\cap y\leqslant\frac{2}{5}\right\}, y\leqslant\frac{2}{5}$ and $y\geqslant-\frac{3}{5}$<br>Accept $\frac{2}{5}$ (or equivalent) and follow through on their $-\frac{3}{5}$ (or equivalent)<br>Do not allow $x$ for the range but allow g or g($x$) but **not** f or f($x$) | | A1ft |
| | | | **(2)** |
| | | | **[11 marks]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **9(a)** | $\sin(2x+x)=\sin 2x\cos x+\cos 2x\sin x$<br>Attempts to use the identity for $\sin(A+B)$ with $A=2x, B=x$ or vice versa<br>Accept $\sin(2x+x)=\sin 2x\cos x\pm\cos 2x\sin x$ or<br>$\sin(x+2x)=\sin x\cos 2x\pm\cos x\sin 2x$ | | M1 |
| | $=2\sin x\cos x\cos x+\left(1-2\sin^2 x\right)\sin x$ | Uses the correct double angle identities for $\sin 2x$ and $\cos 2x$. Allow<br>$\sin 2x=\sin x\cos x+\cos x\sin x$<br>If $\cos 2x=\cos^2 x-\sin^2 x$ is used, then the "$\cos^2 x$" term must be changed to $1-\sin^2 x$ later in the solution. **Dependent on the first method mark.** | **d**M1 |
| | $=2\sin x\left(1-\sin^2 x\right)+\left(1-2\sin^2 x\right)\sin x$ | Reaches an expression in terms of $\sin x$ only by use of<br>$\cos^2 x=1-\sin^2 x$ | M1 |
| | $=3\sin x-4\sin^3 x$ | cso $\sin 3x\equiv 3\sin x-4\sin^3 x$<br>or $\sin 3x\equiv 3\sin x+-4\sin^3 x$ | A1 |
| | **Note: As this is not a "traditional" identity proof with the answer fully given, do not be overly concerned with minor notational errors such as e.g. cos*x*² rather than cos²x or the odd missing "*x*".**<br>**Generally if all the method marks are scored with no clear errors and** $3\sin x-4\sin^3 x$ **is reached, award full marks.** | | |
| | | | **(4)** |

**Part (b) is hence and so they must use part (a) to score in (b)**

| | | | |
|---|---|---|---|
| **9(b)** | $\int \sin 3x\cos x\,dx = \int \left(P\sin x\cos x - Q\sin^3 x\cos x\right)dx$<br>**AND one of:**<br>$\int P\sin x\cos x\,dx = k\sin^2 x$ or $k\cos^2 x$ or $k\cos 2x$<br>**or**<br>$\int Q\sin^3 x\cos x\,dx = k\sin^4 x$<br>**or**<br>$\int Q\sin^3 x\cos x\,dx = \alpha\cos 2x + \beta\cos 4x$<br>(From $4\sin^3 x\cos x = 2\sin^2 x\sin 2x = (1-\cos 2x)\sin 2x = \sin 2x - \frac{1}{2}\sin 4x$) | | M1 |
| | **Examples:**<br>• $= \frac{3}{2}\sin^2 x - \sin^4 x(+c)$<br>• $-\frac{3}{2}\cos^2 x - \sin^4 x(+c)$<br>• $-\frac{3}{4}\cos 2x - \sin^4 x(+c)$<br>• $= \frac{3}{2}\sin^2 x + \frac{1}{2}\cos 2x - \frac{1}{8}\cos 4x$<br>• $-\frac{3}{2}\cos^2 x + \frac{1}{2}\cos 2x - \frac{1}{8}\cos 4x(+c)$<br>• $-\frac{3}{4}\cos 2x + \frac{1}{2}\cos 2x - \frac{1}{8}\cos 4x(+c)$<br>• $-\frac{1}{4}\cos 2x - \frac{1}{8}\cos 4x(+c)$<br>Correct integration | | A1 |
| | E.g. $\left[\frac{3}{2}\sin^2 x - \sin^4 x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{3}{2}\sin^2\left(\frac{\pi}{2}\right) - \sin^4\left(\frac{\pi}{2}\right) - \left\{\frac{3}{2}\sin^2\left(\frac{\pi}{6}\right) - \sin^4\left(\frac{\pi}{6}\right)\right\}$<br>Substitutes both $x = \frac{\pi}{2}$ and $x = \frac{\pi}{6}$ and subtracts either way round<br>**Dependent upon the previous Method mark.** | | **d**M1 |
| | $= \frac{1}{2} - \frac{5}{16} = \frac{3}{16}$ | $\frac{3}{16}$ or 0.1875 (or exact equivalent) | A1 |
| | | | **(4)** |

| | | | |
|---|---|---|---|
| | **Alternative 1 for (b):** | | |
| | $\int \sin 3x \cos x \, dx = \int (P\sin x - Q\sin^3 x)\cos x \, dx$<br>$u = \sin x \Rightarrow \frac{du}{dx} = \cos x \Rightarrow du = \cos x \, dx$<br>$\int (P\sin x - Q\sin^3 x)\cos x \, dx = \int (Pu - Qu^3)\, du$<br>**AND one of:**<br>$\int Pu \, du = ku^2$ **or** $\int Qu^3 \, du = ku^4$ | | M1 |
| | $= \frac{3}{2}u^2 - u^4 (+c)$ | Correct integration | A1 |
| | $\left[\frac{3}{2}u^2 - u^4\right]_{\frac{1}{2}}^{1} = \frac{3}{2} - 1 - \left(\frac{3}{8} - \frac{1}{16}\right)$<br>Substitutes both $x = 1$ and $x = \frac{1}{2}$ and subtracts or replaces $u$ with $\sin x$ and substitutes both $x = \frac{\pi}{2}$ and $x = \frac{\pi}{6}$ and subtracts either way round<br>**Dependent upon the previous Method mark.** | | **d**M1 |
| | $= \frac{1}{2} - \frac{5}{16} = \frac{3}{16}$ | $\frac{3}{16}$ or 0.1875 (or exact equivalent) | A1 |

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| | **Alternative 2 for (b):** | | |
| | $\int \sin 3x \cos x \, dx = \int \left(P\sin x - Q\sin^3 x\right)\cos x \, dx$<br>$= \left(P\sin x - Q\sin^3 x\right)\sin x - \int \left(P\cos x - 3Q\sin^2 x\cos x\right)\sin x \, du$<br>**Parts in the correct direction AND one of:**<br>$\int P\sin x\cos x \, dx = k\sin^2 x$ or $k\cos^2 x$ or $k\cos 2x$<br>**or**<br>$\int Q\sin^3 x\cos x \, dx = k\sin^4 x$<br>**or**<br>$\int Q\sin^3 x\cos x \, dx = \alpha\cos 2x + \beta\cos 4x$ | | M1 |
| | $= 3\sin^2 x - 4\sin^4 x - \frac{3}{2}\sin^2 x + 3\sin^4 x (+c)$<br>Correct integration | | A1 |
| | E.g. $\left[\frac{3}{2}\sin^2 x - \sin^4 x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{3}{2}\sin^2\left(\frac{\pi}{2}\right) - \sin^4\left(\frac{\pi}{2}\right) - \left\{\frac{3}{2}\sin^2\left(\frac{\pi}{6}\right) - \sin^4\left(\frac{\pi}{6}\right)\right\}$<br>Substitutes both $x = \frac{\pi}{2}$ and $x = \frac{\pi}{6}$ and subtracts either way round<br>**Dependent upon the previous Method mark.** | | **d**M1 |
| | $= \frac{1}{2} - \frac{5}{16} = \frac{3}{16}$ | $\frac{3}{16}$ or 0.1875 (or exact equivalent) | A1 |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **10(a)** | $\frac{1}{(2+3x)^3}=(2+3x)^{-3}=\frac{1}{8}\left(1+\frac{3}{2}x\right)^{-3}$ | Takes out a factor of $2^{-3}$ or $\frac{1}{8}$ or $\frac{1}{2^3}$ (or 0.125) | B1 |
| | $\left(1+\frac{3}{2}x\right)^{-3}=1+(-3)\left(\frac{3}{2}x\right)+\frac{(-3)(-4)}{2!}\left(\frac{3}{2}x\right)^2+...$<br>M1: Expands $(1+kx)^{-3}$, $k\neq\pm1$ with the correct structure for the second or third term e.g. $(-3)kx$ or $\frac{(-3)(-4)}{2}(kx)^2$ with or without the bracket around the $kx$<br>Do **not** allow e.g. $\begin{pmatrix}-3\\1\end{pmatrix},\begin{pmatrix}-3\\2\end{pmatrix}$ for the coefficients unless the correct calculations/values are implied by subsequent work | | M1 |
| | $1+(-3)\left(\frac{3}{2}x\right)+\frac{(-3)(-4)}{2!}\left(\frac{3}{2}x\right)^2+..$ | Correct and unsimplified binomial expansion excluding the $\left\{\frac{1}{8}\right\}$ | A1 |
| | $=\frac{1}{8}-\frac{9}{16}x:+\frac{27}{16}x^2$ | $\frac{1}{8}-\frac{9}{16}x$ | A1 |
| | | $\frac{27}{16}x^2$ | A1 |
| | **Special Case – if all the working is correct but the brackets not removed e.g.**<br>$=\frac{1}{8}\left(1-\frac{9}{2}x+\frac{27}{2}x^2\right)$<br>**Score B1M1A1A1A0** | | |
| | | | **(5)** |
| **(a) Way 2** | $(2+3x)^{-3}=2^{-3}+(-3)\times2^{-4}\times(3x)+\frac{(-3)(-4)}{2}\times2^{-5}\times(3x)^2$<br>B1: For first term $2^{-3}$<br>M1: Correct structure for one of the other 2 terms<br>A1: Correct and unsimplified binomial expansion | | B1<br>M1<br>A1 |
| | $=\frac{1}{8}-\frac{9}{16}x:+\frac{27}{16}x^2$ | $\frac{1}{8}-\frac{9}{16}x$ | A1 |
| | | $\frac{27}{16}x^2$ | A1 |

| | | | |
|---|---|---|---|
| **10(b)(i)** | $4\times"\frac{27}{16}"=...$ | $4\times$Their $\frac{27}{16}$ | M1 |
| | Or may start again to expand including their $\frac{1}{8}$: $$\left(2+6x\right)^{-3}=\left\{\frac{1}{8}\right\}\left(1+3x\right)^{-3}=\left\{\frac{1}{8}\right\}\left(1+\left(-3\right)\left(3x\right)+\frac{(-3)(-4)}{2!}\left(3x\right)^{2}+...\right)$$ or $$\left(2+6x\right)^{-3}=2^{-3}+(-3)\times2^{-4}\times(6x)+\frac{(-3)(-4)}{2}\times2^{-5}\times(6x)^{2}$$ Or uses their (possibly incorrect) expansion from (a) with $3x$ instead of $\frac{3}{2}x$ <br>**<u>And evaluates the coefficient of their $x^2$ term</u>** | | |
| | $\frac{27}{4}$ | Allow exact equivalents e.g. 6.75, $6\frac{3}{4}$. **Must be seen identified as the required term and not just part of an expansion.** | A1 |
| **(b)(ii)** | $4\times"\frac{27}{16}"-\left("-\frac{9}{16}"\right)=...$ | $4\times$Their $\frac{27}{16}\pm$Their$-\frac{9}{16}$... | M1 |
| | **If the candidate attempts a complete expansion, this mark can score as long as the $x^2$ terms are collected** | | |
| | $\frac{117}{16}$ | Allow exact equivalents e.g. 7.3125, $7\frac{5}{16}$. **Must be seen identified as the required term and not just part of an expansion.** | A1 |
| | **Special Case:** <br>**If the $x^2$ s are included with the coefficients then penalise this once only and at the first occurrence.** | | **(4)** |
| | | | |
| | | | **[9 marks]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **11(a)** | $$\frac{9}{t^2(t-3)}=\frac{A}{t}+\frac{B}{t^2}+\frac{C}{t-3}$$ | | |
| | $9=At(t-3)+B(t-3)+Ct^2$ | A correct equation. (May be implied) | B1 |
| | $t=3\Rightarrow C=...$ or $t=0\Rightarrow B=...$<br>or<br>$9=At^2-3At+Bt-3B+Ct^2$<br>$-3B=9\Rightarrow B=...$ | Finds one constant by either substitution or use of simultaneous equations | M1 |
| | $A=-1,\ B=-3,\ C=1$ | Correct values or correct fractions | A1 |
| | | | **(3)** |
| **(b)** | **In part (b), condone the use of *x* rather than *t* and log for ln.** | | |
| | $\int -\frac{1}{t}+\frac{1}{t-3}\mathrm{d}t=-\ln t+\ln(t-3)$ | Allow for $\int \frac{A}{t}+\frac{C}{t-3}\mathrm{d}t=\alpha\ln t+\beta\ln(t-3)$ | M1 |
| | $\int -\frac{3}{t^2}\mathrm{d}t=\frac{3}{t}$ | Allow for $\int \frac{B}{t^2}\mathrm{d}t=\pm\frac{\alpha}{t}$ | M1 |
| | $\int \frac{9}{t^2(t-3)}\mathrm{d}t=\int\left(-\frac{1}{t}-\frac{3}{t^2}+\frac{1}{t-3}\right)\mathrm{d}t=-\ln t+\frac{3}{t}+\ln(t-3)+(c)$<br>Correct integration (possibly unsimplified) or correct follow through (possibly unsimplified) for their non-zero *A*, *B* and *C* e.g.<br>$\int\left(\frac{A}{t}+\frac{B}{t^2}+\frac{C}{t-3}\right)\mathrm{d}t=A\ln t-\frac{B}{t}+C\ln(t-3)$ | | A1ft |
| | $I=\left[-\ln t+\frac{3}{t}+\ln(t-3)+(c)\right]_4^{12}=\left(-\ln 12+\frac{3}{12}+\ln 9\right)-\left(-\ln 4+\frac{3}{4}+\ln 1\right)$<br>For substituting in 12 and 4 into a “changed” function and subtracting either way round – may be implied by their values | | M1 |
| | $=\ln\left(\frac{9\times 4}{12}\right)-\frac{1}{2}$<br>**Dependent on all the previous method marks.**<br>Must be <u>fully correct</u> log work for their values to combine the ln’s into a single logarithm. Note that some candidates combine their logs before substitution and this mark can score then for fully correct log work. | | **ddd**M1 |
| | $=\ln(3)-\frac{1}{2}$ | Cso. Condone lack of brackets. Allow equivalents for the $\frac{1}{2}$ e.g. 0.5 or $\frac{2}{4}$ | A1 |
| | | | **(6)** |

| | | | |
|---|---|---|---|
| **11(c)** | $x = 2\ln(t-3) \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{2}{t-3}$ | Correct expression for $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ (may be implied) | B1 **M1 on ePEN** |
| | $V = \int \pi y^2 \dfrac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t = \int \pi \times \dfrac{36}{t^2} \times \dfrac{2}{(t-3)}\mathrm{d}t$<br>Uses $(\pi\times)\int y^2 \dfrac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t = \int \left(\dfrac{6}{t}\right)^2 \times \mathit{their}\, \dfrac{2}{(t-3)}\mathrm{d}t$<br>Condone missing brackets, missing π and missing d$t$ | | M1 |
| | $= 8\pi \times I$<br>Correct volume in terms of π. Allow $k$ = 8π.<br>For this mark to be awarded there must be reference to the limits at some stage e.g. shows $x = 0 \Rightarrow t = 4$ and $x = 2\ln 9 \Rightarrow t = 12$ or starts with an integral with limits 0 and 2ln9 and changes to limits 4 and 12 | | A1 |
| | **Ignore subsequent attempts to evaluate the integral but the A1 can be awarded for e.g.** $V = 8\pi\left(\ln 3 - \dfrac{1}{2}\right)$ **provided the above conditions for the A1 are also met.** | | |
| | | | **(3)** |
| | | | **[12 marks]** |

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="3">12(a)</td><td colspan="2">$\overrightarrow{AB}=(3\mathbf{i}+2\mathbf{j}+5\mathbf{k})-(2\mathbf{i}-3\mathbf{j}-2\mathbf{k})=...$<br>For subtracting either way around. Accept $\pm\left((3\mathbf{i}+2\mathbf{j}+5\mathbf{k})-(2\mathbf{i}-3\mathbf{j}-2\mathbf{k})\right)$<br>It may be implied by two out of three terms correct</td><td>M1</td></tr>
<tr><td>$\overrightarrow{AB}=(\mathbf{i}+5\mathbf{j}+7\mathbf{k})$</td><td>$\overrightarrow{AB}=(\mathbf{i}+5\mathbf{j}+7\mathbf{k})$ or the equivalent column vector but not the coordinates and not $\begin{pmatrix}\mathbf{i}\\5\mathbf{j}\\7\mathbf{k}\end{pmatrix}$</td><td>A1</td></tr>
<tr><td></td><td></td><td>(2)</td></tr>
<tr><td rowspan="3">(b)</td><td rowspan="2">Examples:<br>$\boldsymbol{r}=(2\mathbf{i}-3\mathbf{j}-2\mathbf{k})+\lambda(\mathbf{i}+5\mathbf{j}+7\mathbf{k})$<br>$\boldsymbol{r}=(2\mathbf{i}-3\mathbf{j}-2\mathbf{k})-\lambda(\mathbf{i}+5\mathbf{j}+7\mathbf{k})$<br>$\boldsymbol{r}=(3\mathbf{i}+2\mathbf{j}+5\mathbf{k})+\lambda(\mathbf{i}+5\mathbf{j}+7\mathbf{k})$<br>$\boldsymbol{r}=(3\mathbf{i}+2\mathbf{j}+5\mathbf{k})-\lambda(\mathbf{i}+5\mathbf{j}+7\mathbf{k})$</td><td>Correct method for l. Needs a point on l (usually A or B) ±"λ" their (a) or an attempt at $\pm\overrightarrow{AB}$.</td><td>M1</td></tr>
<tr><td>$\boldsymbol{r}=\overrightarrow{OA}\pm\lambda\left(\overrightarrow{AB}\right)$ or $\boldsymbol{r}=\overrightarrow{OB}\pm\lambda\left(\overrightarrow{AB}\right)$<br>or equivalent e.g. column vectors but not with i , j, k within the columns as above. A fully correct equation <u>including</u> "r ="</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>(2)</td></tr>
<tr><td rowspan="5">(c)</td><td colspan="2">$\overrightarrow{AC}=(2\mathbf{i}+4\mathbf{j}-3\mathbf{k})-(2\mathbf{i}-3\mathbf{j}-2\mathbf{k})=(0\mathbf{i}+7\mathbf{j}-1\mathbf{k})$<br>For using vector $\overrightarrow{OA}$ and $\overrightarrow{OC}$ and subtracting either way around<br>$\overrightarrow{AC}=\pm\left((2\mathbf{i}+4\mathbf{j}-3\mathbf{k})-(2\mathbf{i}-3\mathbf{j}-2\mathbf{k})\right)$<br>It may be implied by two out of three terms correct</td><td>M1</td></tr>
<tr><td colspan="2">Attempts $\overrightarrow{AB}.\overrightarrow{AC}=\lvert AB\rvert\lvert AC\rvert\cos\theta\Rightarrow 0+35-7=\sqrt{75}\sqrt{50}\cos\theta$<br>For an attempt at $\overrightarrow{AB}.\overrightarrow{AC}=\lvert AB\rvert\lvert AC\rvert\cos\theta$<br>with "their" $\overrightarrow{AB}$ or $\overrightarrow{BA}$ and "their" $\overrightarrow{AC}$ or $\overrightarrow{CA}$<br>Can be scored for cosine rule with<br>$\cos\theta=\dfrac{\lvert AB\rvert^2+\lvert AC\rvert^2-\lvert CB\rvert^2}{2\lvert AB\rvert\lvert AC\rvert}=\dfrac{75+50-69}{2\sqrt{75}\sqrt{50}}$</td><td>M1</td></tr>
<tr><td colspan="2">$\Rightarrow\cos\theta=\dfrac{28}{\sqrt{75}\sqrt{50}}\left(\text{or }\dfrac{14\sqrt{6}}{75}\right)$ or $\cos\theta=-\dfrac{28}{\sqrt{75}\sqrt{50}}\left(\text{or }-\dfrac{14\sqrt{6}}{75}\right)$<br>Correct value for cos θ (allow awrt ± 0.457)</td><td>A1</td></tr>
<tr><td colspan="2">$\Rightarrow\theta=62.8°$ Cso<br>If the answer of θ = 117.2° is obtained the minimum we would expect for this final mark is $180°-117.2°=62.8°$</td><td>A1*</td></tr>
<tr><td colspan="2"></td><td>(4)</td></tr>
</table>

| | | |
|---|---|---|
| **12(d)** | $Area = \frac{1}{2} their \lvert AB \rvert \times their \lvert AC \rvert \sin(62.8°)$<br>$= \frac{1}{2}\lvert \mathbf{i}+5\mathbf{j}+7\mathbf{k}\rvert\lvert 7\mathbf{j}-\mathbf{k}\rvert \sin(62.8°) = \frac{1}{2}\sqrt{1^2+5^2+7^2}\sqrt{7^2+1^2}\sin(62.8°)$<br>$\frac{1}{2}\sqrt{75}\sqrt{50}\sin(62.8°)$ | M1 |
| | $= 27.2$ — Allow awrt 27.2 | A1 |
| | **Correct answer only scores both marks** | |
| | | **(2)** |
| **(e)** | Method of finding one coordinate or position vector of point *D*.<br>$(\overrightarrow{OD} =)(2\mathbf{i}-3\mathbf{j}-2\mathbf{k}) \pm 2\times(\mathbf{i}+5\mathbf{j}+7\mathbf{k})$<br>**or**<br>$(\overrightarrow{OD} =)(3\mathbf{i}+2\mathbf{j}+5\mathbf{k}) + (\mathbf{i}+5\mathbf{j}+7\mathbf{k})$<br>**or**<br>$(\overrightarrow{OD} =)(3\mathbf{i}+2\mathbf{j}+5\mathbf{k}) - 3\times(\mathbf{i}+5\mathbf{j}+7\mathbf{k})$ | M1 |
| | $\overrightarrow{OD} = (4\mathbf{i}+7\mathbf{j}+12\mathbf{k})$ and $(0\mathbf{i}-13\mathbf{j}-16\mathbf{k})$<br>A1: **One** position vector or **one** set of coordinates correct<br>A1: Both **position vectors** correct<br>**Do not isw and mark their final answers** | A1 A1 |
| | | **(3)** |
| | **Note that there are many ways of answering part (e) which are more convoluted, however, the M mark should be awarded as follows:**<br>$\overrightarrow{OD} = (2\mathbf{i}-3\mathbf{j}-2\mathbf{k}) + \alpha(\mathbf{i}+5\mathbf{j}+7\mathbf{k})$<br>**or**<br>$\overrightarrow{OD} = (2\mathbf{i}-3\mathbf{j}-2\mathbf{k}) - \alpha(\mathbf{i}+5\mathbf{j}+7\mathbf{k})$<br>Where $1.99 < \alpha < 2.01$<br>**or**<br>$\overrightarrow{OD} = (3\mathbf{i}+2\mathbf{j}+5\mathbf{k}) + \beta(\mathbf{i}+5\mathbf{j}+7\mathbf{k})$<br>Where $0.99 < \beta < 1.01$<br>**or**<br>$\overrightarrow{OD} = (3\mathbf{i}+2\mathbf{j}+5\mathbf{k}) - \gamma(\mathbf{i}+5\mathbf{j}+7\mathbf{k})$<br>Where $2.99 < \gamma < 3.01$ | |
| | | **[13 marks]** |

| Question Number | Scheme | | Marks |
| --- | --- | --- | --- |
| **13 (a)** | $h = 0.25$ | Correct strip width | B1 |
| | Area = $\frac{0.25}{2}\left\{8.32 + 99.8 + 2\times\left(21.4 + 40.6 + 66.6\right)\right\}$<br>Correct trapezium rule structure e.g.<br>$\frac{h}{2}\{\text{Correct } y\text{-value structure}\}$<br>Or may see separate trapezia:<br>$\frac{0.25}{2}(8.32+21.4)+\frac{0.25}{2}(21.4+40.6)+\frac{0.25}{2}(40.6+66.6)+\frac{0.25}{2}(66.6+99.8)$ | | M1 |
| | Awrt 46 | | A1 |
| | A correct answer of awrt 46 with no incorrect working seen can score full marks. Note that calculator gives 45.1028… | | |
| | | | **(3)** |
| **(b)** | $u = x^2 \Rightarrow \frac{du}{dx} = 2x$ | $u = x^2 \Rightarrow \frac{du}{dx} = 2x$ or equivalent<br>such as $x = u^{\frac{1}{2}} \Rightarrow \frac{dx}{du} = \frac{1}{2}u^{-\frac{1}{2}}$ | B1 |
| | Area $R$ = $\int 12x^2 \ln\left(2x^2\right)dx = \int 12u\ln\left(2u\right)\frac{1}{2}u^{-\frac{1}{2}}\,du$<br>or<br>$\int 12x^2\ln\left(2x^2\right)dx = \int 12u\ln\left(2u\right)\frac{du}{2x}$<br>or<br>$\int 12x^2\ln\left(2x^2\right)dx = \int 12x^2\ln\left(2u\right)\frac{du}{2x}$<br>or<br>$\int 12x^2\ln\left(2x^2\right)dx = \int 12x^2\ln\left(2u\right)\frac{1}{2}u^{-\frac{1}{2}}\,du$<br>Uses the substitution and replaces at least the "d$x$" in terms of d$u$ **and** changes the ln($2x^2$) to ln($2u$) | | M1 |
| | $= \int_1^4 6u^{\frac{1}{2}}\ln\left(2u\right)\,du$ *<br>Completes to obtain the printed answer. There must be a reference to the limits e.g. clear evidence of the change of limits or with the 1 and 2 in the $x$ integral becoming 1 and 4 in the $u$ integral.<br>Allow working to appear with e.g. integral signs missing but if limits are attached at any stage they must correspond with the "d$x$" or the "d$u$" present at that stage. At some point, the $12x^2$ and the d$x$ must appear in terms of the same variable e.g. as $\frac{12x^2}{2x}$ or as $\frac{12u}{2\sqrt{u}}$. | | A1* |
| | | | **(3)** |

| | | | |
|---|---|---|---|
| **13(b) Way 2** | $u = x^2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$ | $u = x^2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$ or equivalent<br>such as $x = u^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{2}u^{-\frac{1}{2}}$ | B1 |
| | Area $R$ = $\int 6u^{\frac{1}{2}} \ln(2u)\,\mathrm{d}u = \int 6x\ln(2x^2)2x\,\mathrm{d}x$<br>or<br>$\int 6u^{\frac{1}{2}} \ln(2u)\,\mathrm{d}u = \int 6u^{\frac{1}{2}} \ln(2x^2)2x\,\mathrm{d}x$<br>or<br>$\int 6u^{\frac{1}{2}} \ln(2u)\,\mathrm{d}u = \int 6u^{\frac{1}{2}} \ln(2x^2)2\sqrt{u}\,\mathrm{d}x$<br>or<br>$\int 6u^{\frac{1}{2}} \ln(2u)\,\mathrm{d}u = \int 6x\ln(2x^2)2\sqrt{u}\,\mathrm{d}x$<br>Uses the substitution and replaces at least the "d$u$" in terms of d$x$ **and** changes the ln($u$) to ln($2x^2$) | | M1 |
| | $= \int_1^2 12x^2 \ln(2x^2)\,\mathrm{d}x$ *<br>**Which is the area of R.**<br>Completes to obtain the printed answer **<u>with a conclusion</u>**. There must be a reference to the limits e.g. clear evidence of the change of limits or with the 1 and 4 in the $u$ integral becoming 1 and 2 in the $x$ integral.<br>Allow working to appear with e.g. integral signs missing but if limits are attached at any stage they must correspond with the "d$x$" or the "d$u$" present at that stage. At some point, the $6u^{\frac{1}{2}}$ and the d$u$ must appear in terms of the same variable e.g. as $6x \times 2x$ or as $6u^{\frac{1}{2}} \times 2\sqrt{u}$. | | A1* |

| | | |
|---|---|---|
| **13(c)** | $$\int 6u^{\frac{1}{2}}\ln 2u\,\mathrm{d}u = 4u^{\frac{3}{2}}\ln 2u - \int 4u^{\frac{3}{2}}\times\frac{1}{u}\mathrm{d}u$$ M1: Integrates by parts the correct way around achieving $$Pu^{\frac{3}{2}}\ln 2u - \int Qu^{\frac{3}{2}}\times\frac{1}{u}\mathrm{d}u$$ A1: $4u^{\frac{3}{2}}\ln 2u - \int 4u^{\frac{3}{2}}\times\frac{1}{u}\mathrm{d}u$ or exact equivalent **or if the "6" is omitted allow** $\frac{2}{3}u^{\frac{3}{2}}\ln 2u - \int \frac{2}{3}u^{\frac{3}{2}}\times\frac{1}{u}\mathrm{d}u$ or exact equivalent | M1A1 |
| | $$= 4u^{\frac{3}{2}}\ln 2u - \frac{8}{3}u^{\frac{3}{2}}$$ **or if the "6" is omitted allow** $$= \frac{2}{3}u^{\frac{3}{2}}\ln 2u - \frac{4}{9}u^{\frac{3}{2}}$$ | A1 |
| | Area $= \left[4u^{\frac{3}{2}}\ln 2u - \frac{8}{3}u^{\frac{3}{2}}\right]_1^4 = \left(32\ln 8 - \frac{64}{3}\right) - \left(4\ln 2 - \frac{8}{3}\right)$ **Dependent upon the previous M**. It is scored for putting in the limits of 4 and 1 and subtracting either way around. Alternatively they could use the limits of 1 and 2 with a substituted function in $x$. | **d**M1 |
| | $$= 96\ln 2 - 4\ln 2 - \frac{64}{3} + \frac{8}{3} = \alpha\ln 2 + ...$$ For correct log work on their ln8 term and combining correctly with ln2 term to obtain a single ln2 term **having substituted into an integrated function.** | M1 |
| | $= -\frac{56}{3} + 92\ln 2\left(\text{or } 92\ln 2 - \frac{56}{3}\right)$ \| $92\ln 2 - \frac{56}{3}$ or $-\frac{56}{3} + 92\ln 2$ | A1 |
| | | **(6)** |
| | | **[12 marks]** |

<table>
<tr><td rowspan="7">13(c)</td><td colspan="2">Alternative in terms of x:</td><td></td></tr>
<tr><td colspan="2">$$\int 12x^2\ln\left(2x^2\right)\mathrm{d}x=\frac{12x^3}{3}\ln\left(2x^2\right)-\int\frac{12x^3}{3}\times\frac{4x}{2x^2}\,\mathrm{d}x$$
M1: Integrates by parts the correct way around achieving
$$Px^3\ln\left(2x^2\right)-\int Qx^3\times\frac{1}{x}\,\mathrm{d}x$$
A1: $\frac{12x^3}{3}\ln\left(2x^2\right)-\int\frac{12x^3}{3}\times\frac{4x}{2x^2}\,\mathrm{d}x$ or exact equivalent<br>
or if the “12” is omitted allow<br>
$\frac{x^3}{3}\ln\left(2x^2\right)-\int\frac{x^3}{3}\times\frac{4x}{2x^2}\,\mathrm{d}x$ or exact equivalent</td><td>M1A1</td></tr>
<tr><td colspan="2">$$=4x^3\ln\left(2x^2\right)-\frac{8}{3}x^3$$
or if the “12” is omitted allow
$$=\frac{1}{3}x^3\ln\left(2x^2\right)-\frac{2}{9}x^3$$</td><td>A1</td></tr>
<tr><td colspan="2">$$\text{Area }=\left[4x^3\ln\left(2x^2\right)-\frac{8}{3}x^3\right]_1^2=\left(32\ln 8-\frac{64}{3}\right)-\left(4\ln 2-\frac{8}{3}\right)$$
Dependent upon the previous M. It is scored for putting in the limits of 2 and 1 and subtracting either way around.</td><td>dM1</td></tr>
<tr><td colspan="2">$$=96\ln 2-4\ln 2-\frac{64}{3}+\frac{8}{3}$$
For correct log work on the ln8 term and combining correctly with ln2 term to obtain a single ln2 term having substituted into an integrated function.</td><td>M1</td></tr>
<tr><td>$=92\ln 2-\frac{56}{3}$</td><td>$92\ln 2-\frac{56}{3}$ CSO</td><td>A1</td></tr>
</table>

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 14(a) | **Allow $e^1$ for e throughout.** | | |
| | $9$ or $3+\mathrm{e}$ | For sight of either intercept $9$ (not $10-\mathrm{e}^0$) or $3+\mathrm{e}$ or $3+\mathrm{e}^1$ or $3+\mathrm{e}^{0+1}$ | M1 |
| | Distance $PQ=6-\mathrm{e}$ | $6-\mathrm{e}$ (Not $9-(3+\mathrm{e})$) | A1 |
| | | | **(2)** |
| (b) | Sets $3+\mathrm{e}^{x+1}=10-\mathrm{e}^x$ | Equates the 2 curves | M1 |
| | $\mathrm{e}^x(\mathrm{e}+1)=7$ | Collects exponential terms and takes out a factor of $\mathrm{e}^x$ with correct index work. | M1 |
| | $x=\ln\left(\dfrac{7}{1+\mathrm{e}}\right)$ or $\ln\dfrac{7}{1+\mathrm{e}}$ | Correct $x$-coordinate | A1 |
| | Substitutes their $x=\ln\left(\dfrac{7}{1+\mathrm{e}}\right)$ in $y=10-\mathrm{e}^x\Rightarrow y=..$<br>**Dependent upon both M's.**<br>It is for substituting their value of $x$ into either equation to find $y$ | | **dd**M1 |
| | $R=\left(\ln\left(\dfrac{7}{1+\mathrm{e}}\right),\dfrac{3+10\mathrm{e}}{1+\mathrm{e}}\right)$ | $R=\left(\ln\left(\dfrac{7}{1+\mathrm{e}}\right),\dfrac{3+10\mathrm{e}}{1+\mathrm{e}}\right)$ or<br>equivalent such as<br>$x=\ln\left(\dfrac{7}{1+\mathrm{e}}\right),\ y=10-\dfrac{7}{1+\mathrm{e}}$ or<br>$y=3+\dfrac{7\mathrm{e}}{1+\mathrm{e}}$ but **not** $y=10-\mathrm{e}^{\ln\frac{7}{1+\mathrm{e}}}$ | A1 |
| | | | **(5)** |
| (b) **Way 2** | $y=10-\mathrm{e}^x\Rightarrow\mathrm{e}^x=10-y\Rightarrow x=\ln(10-y)\Rightarrow y=3+\mathrm{e}^{1+\ln(10-y)}$<br>Makes $x$ the subject of equation 2 and substitutes into equation 1 | | M1 |
| | $y=3+(10-y)\mathrm{e}$ | Uses correct index work to eliminate the "ln" | M1 |
| | $\Rightarrow y=\dfrac{10\mathrm{e}+3}{1+\mathrm{e}}\left(\text{or } 10-\dfrac{7}{1+\mathrm{e}}\right)$<br>or $y=3+\dfrac{7\mathrm{e}}{1+\mathrm{e}}$ | Correct $y$-coordinate.<br>**Not** $y=10-\mathrm{e}^{\ln\frac{7}{1+\mathrm{e}}}$ | A1 |
| | $x=\ln(10-y)=\ln\left(10-\dfrac{10\mathrm{e}+3}{1+\mathrm{e}}\right)$ | Dependent upon both M's. It is for substituting their value of $y$ into either equation to find $x$.<br>**Dependent upon both M's.** | **dd**M1 |
| | $R=\left(\ln\left(\dfrac{7}{1+\mathrm{e}}\right),\dfrac{3+10\mathrm{e}}{1+\mathrm{e}}\right)$ | $R=\left(\ln\left(\dfrac{7}{1+\mathrm{e}}\right),\dfrac{3+10\mathrm{e}}{1+\mathrm{e}}\right)$ or<br>equivalent such as<br>$x=\ln\left(\dfrac{7}{1+\mathrm{e}}\right),\ y=10-\dfrac{7}{1+\mathrm{e}}$<br>Allow $x=\ln\left(10-\dfrac{10\mathrm{e}+3}{1+\mathrm{e}}\right)$ | A1 |

| | | | |
|---|---|---|---|
| **14(b) Way 3** | $y=3+e^{x+1} \Rightarrow y-3=e^{x+1} \Rightarrow e^x=\frac{y-3}{e} \Rightarrow 10-y=\frac{y-3}{e}$<br>Makes $e^x$ the subject of equation 1 and substitutes into equation 2 | | M1 |
| | $10-y=\frac{y-3}{e} \Rightarrow 10e-ye=y-3 \Rightarrow y(1+e)=10e+3$<br>Uses correct algebra and factorises $y$ | | M1 |
| | $\Rightarrow y=\frac{10e+3}{1+e}\left(\text{or } 10-\frac{7}{1+e}\right)$<br>or $y=3+\frac{7e}{1+e}$ | Correct $y$-coordinate | A1 |
| | **Then as above.** | | |
| **(b) Way 4** | $y=10-e^x \Rightarrow e^x=10-y \Rightarrow x=\ln(10-y)$<br>$y=3+e^{x+1} \Rightarrow e^{x+1}=y-3 \Rightarrow x+1=\ln(y-3)$<br>$y=3+e^{x+1} \Rightarrow e^{x+1}=y-3 \Rightarrow \ln(10-y)=\ln(y-3)-1$<br>Makes $x$ and $x+1$ the subject and uses $x=x$ | | M1 |
| | $\Rightarrow \frac{y-3}{(10-y)}=e$ | Uses correct work to eliminate the "ln's" | M1 |
| | $\Rightarrow y=\frac{10e+3}{1+e}\left(\text{or } 10-\frac{7}{1+e}\right)$<br>or $y=3+\frac{7e}{1+e}$ | Correct $y$-coordinate | A1 |
| | **Then as above.** | | |
| | | | **[7 marks]** |

**If the candidate works in decimals throughout then the <u>method</u> marks are still available if no exact values are seen. NB: 3 + e = 5.71…, 6 – e = 3.28…, $\ln\left(\frac{7}{1+e}\right)$ = 0.632…, $\frac{10e+3}{1+e}$ = 8.11…**

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