Mark Scheme (Results)

January 2018

Pearson Edexcel
International Advanced Subsidiary Level
In Core Mathematics C34 (WMA02)
Paper 01

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January 2018
Publications Code WMA02_01_1801_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - ⎿ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1	Differentiates wrt x $\quad 3^x \ln 3 + x\dfrac{dy}{dx} + y = 1 + 2y\dfrac{dy}{dx}$	B1, B1, M1, A1
	Substitutes (4, 11) **AND** rearranges to get $\dfrac{dy}{dx} = ..$ $\quad$ Nb $\dfrac{dy}{dx} = \dfrac{3^x \ln 3 + y - 1}{2y - x}$	M1
	$\Rightarrow 81\ln 3 + 4\dfrac{dy}{dx} + 11 = 1 + 22\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{81\ln 3 + 10}{18} = \dfrac{5}{9} + \dfrac{9}{2}\ln 3$	A1
		(6)
		(6 marks)

B1 $\quad 3^x \to 3^x \ln 3$ or $e^{x\ln 3} \to e^{x\ln 3}\ln 3$

B1 $\quad$ Correct product rule to differentiate xy finding $x\dfrac{dy}{dx} + y$

This may appear as $x\,dy + y\,dx$

M1 $\quad$ Differentiates implicitly to get $y^2 \to ky\dfrac{dy}{dx}$

This may appear as $y^2 \to k\,y\,dy$

A1 $\quad$ A correct differential of all terms other than 3^x so $"their\, 3^x \ln 3" + x\dfrac{dy}{dx} + y = 1 + 2y\dfrac{dy}{dx}$

If an extra $\dfrac{dy}{dx}$ is seen, ie $\dfrac{dy}{dx} = "their\, 3^x \ln 3" + x\dfrac{dy}{dx} + y = 1 + 2y\dfrac{dy}{dx}$ you may allow recovery if it is subsequently ignored.

You may see this as $"their\, 3^x \ln 3"dx + x\,dy + y\,dx = 1dx + 2y\,dy$

M1 $\quad$ Substitutes both $x = 4,\ y = 11$ into their expression (seen or implied at least once) and finds a 'numerical' value for $\dfrac{dy}{dx}$ (may rearrange first to give $\dfrac{dy}{dx} =$).

It is dependent upon having two terms in $\dfrac{dy}{dx}$ and proceeding, condoning slips, to $\dfrac{dy}{dx} = ...$

A1 $\quad$ Exact answer only but accept any equivalent e.g. $\dfrac{10}{18} + \dfrac{81}{18}\ln 3,\ \dfrac{5}{9} + 4.5\ln 3$

Remember to isw after sight of the correct answer

Note: If a candidate finds the equation of the tangent without specifically stating $\dfrac{dy}{dx} = ..$ they can score the M mark but not the A. If they give $\dfrac{dy}{dx} = ..$ you can apply isw

Question Number	Scheme		Marks
2 (a)	$\text{f}(x)=(125-5x)^{\frac{2}{3}}=125^{\frac{2}{3}}(1-\frac{1}{25}x)^{\frac{2}{3}}$	$125^{\frac{2}{3}}$ or 25	B1
	$=25\times\underline{\left(1+\left(\frac{2}{3}\right)\left(-\frac{1}{25}x\right)+\frac{\left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)}{2}\left(-\frac{1}{25}x\right)^2+\ldots\right)}$		<u>M1A1</u>
	$=25-\frac{2}{3}x-\frac{1}{225}x^2\ldots$		A1 **(4)**
	Alternative: $(125-5x)^{\frac{2}{3}}=125^{\frac{2}{3}}+\left(\frac{2}{3}\right)125^{-\frac{1}{3}}(-5x)+\frac{\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)}{2}125^{-\frac{4}{3}}(-5x)^2$		B1 M1 A1
	$=25-\frac{2}{3}x-\frac{1}{225}x^2\ldots$		A1 **(4)**
(b)	Let $x=1$ Evaluate $=25-\frac{2}{3}-\frac{1}{225}$ = 24.32889		B1 M1 A1 **(3)**
			(7 marks)

(a)

Way 1:

B1 For taking out a factor of $125^{\frac{2}{3}}$ or 25

M1 For the form of the binomial expansion with $n=\frac{2}{3}$ and a term of $\left(\pm\frac{1}{25}x\right)$, $\left(\pm\frac{5}{125}x\right)$ or $\left(\pm\left(\frac{1}{5}\right)^2x\right)$

To score M1 it is sufficient to see either term two or term three. Allow a slip on the sign of $\left(-\frac{1}{25}x\right)$. So allow for either $\left(\frac{2}{3}\right)\left(\pm\frac{1}{25}x\right)$ or $\frac{\left(\frac{2}{3}\right)\times\left(-\frac{1}{3}\right)}{2}\left(\pm\frac{1}{25}x\right)^2$

A1 Any (unsimplified) form of the binomial expansion. Ignore factor preceding the bracket

A1 cao $=25-\frac{2}{3}x-\frac{1}{225}x^2\ldots$ This must be simplified. Ignore extra terms.

(a) Way 2:

B1 For seeing either $125^{\frac{2}{3}}$ or 25 as the first term

M1 It is sufficient to see either term two or term three (unsimplified or simplified).

Allow a slip on the sign of $(-5x)$

So allow for either $\left(\frac{2}{3}\right)125^{-\frac{1}{3}}(\pm 5x)$ or $\frac{\left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)}{2}125^{-\frac{4}{3}}(\pm 5x)^2$

The expression $125^{2/3}+\binom{2/3}{1}125^{-\frac{1}{3}}(\pm 5x)+\binom{2/3}{2}125^{-\frac{4}{3}}(\pm 5x)^2+$

does not score the method mark until one of the terms is processed as in the main method

A1 Any (un-simplified) form of the whole binomial expansion.

A1 Must now be simplified cao $=25-\frac{2}{3}x-\frac{1}{225}x^2-...$

(b)

B1: States $x = 1$or is **explicitly seen** to use $x = 1$

M1: See an attempt to substitute a value of x consistently in **their** series expansion, condoning slips.

May be implied by sight of $=25-\frac{2}{3}-\frac{1}{225}$ or the correct answer for their expression.

Allow if they have more terms, but not if they have fewer.

A1: cao 24.32889 * DO NOT ACCEPT AWRT*

Watch: 24.32881 is the calculator answer for $120^{2/3}$

Note: If there is a decimal answer and they don't show their method you will need to use your calculator with $x = 1$ to check their result for the M1

Correct part (a)

Eg 1. (a) $25-\frac{2}{3}x-\frac{1}{225}x^2-...$ (b) $\Rightarrow 120^{2/3}=24.32889$ B0(not stated or seen) M1(implied) A1

Examples 2 to 5: Incorrect part (a)

Eg 2.(a) $25-\frac{2}{3}x+\frac{1}{225}x^2-...$ (b) $120^{2/3}=24.33778$ B0 M1(implied by calculator check) A0

Eg 3.(a) $25-\frac{2}{3}x+\frac{1}{225}x^2-...$ (b) $x = 1 \Rightarrow 120^{2/3}=24.33778$ B1 (stated) M1(implied by calculator check) A0

Eg 4.(a) $25-\frac{2}{3}x+\frac{1}{225}x^2-...$ (b) $\Rightarrow 120^{2/3}=25-\frac{2}{3}\times 1+\frac{1}{225}\times 1^2=24.33778$ B1(seen) M1(seen) A0

Eg 5.(a) $25-\frac{2}{3}x+\frac{1}{225}x^2-...$ (b) $\Rightarrow 120^{2/3}=25-\frac{2}{3}+\frac{1}{225}=24.33778$ B0(not stated or seen) M1(implied) A0

Question Number	Scheme	Marks
3. (a)	$\mathrm{f}(x)=\frac{x^2}{4}+\ln(2x)=0$ so $\ln(2x)=-\frac{x^2}{4}$ and so $2x=\mathrm{e}^{-\frac{x^2}{4}}$ and $x=\frac{1}{2}\mathrm{e}^{-\frac{x^2}{4}}$ *	M1 A1* **(2)**
(b)	$x_2=\frac{1}{2}\mathrm{e}^{-\frac{(0.5)^2}{4}}$	M1
	$x_2=\text{awrt } 0.4697$,	A1
	$x_3=\text{awrt } 0.4732$ and $x_4=\text{awrt } 0.4728$	A1 **(3)**
(c)	$\mathrm{f}(0.4725)=-0.000756...<0$, $\mathrm{f}(0.4735)=0.001594..>0$ Sign change (and as $\mathrm{f}(x)$ is continuous) therefore root lies in the interval $[0.4725, 0.4735] \Rightarrow root=0.473$ (3 dp)	M1A1 **(2)**
		(7 marks)

(a)

M1: Puts f(x) = 0, either stated, or implied by sight of $\frac{x^2}{4}+\ln(2x)=0$, then makes the (2x) of ln(2x) the subject by taking the exponential. Condone slips and the omission of the bracket (very common) but taking the exp of each term, $\frac{x^2}{4}+\ln(2x)=0\Rightarrow \mathrm{e}^{\frac{x^2}{4}}+2x=0$ is M0

Alternatively, award for a correct answer with a missing first step $-\frac{x^2}{4}=\ln(2x)\Rightarrow 2x=\mathrm{e}^{-\frac{x^2}{4}}\Rightarrow x=\frac{1}{2}\mathrm{e}^{-\frac{x^2}{4}}$

Candidates who work backwards must proceed from $x=\frac{1}{2}\mathrm{e}^{-\frac{x^2}{4}}$ to $\frac{x^2}{4}+\ln(2x)=0$ before the M mark is scored. They need to make a comment before the A mark is awarded. Eg. Hence f(x) = 0

A1*: Completely correct work **ignoring** bracketing on $\ln(2x)$ to achieve the printed answer.

(b)

M1: An attempt to substitute $x_1=0.5$ into the iterative formula

This may be implied by sight of $x_2=\frac{1}{2}\mathrm{e}^{-\frac{(0.5)^2}{4}}$ $x_2=awrt\ 0.47$

A1: $x_2=awrt\ 0.4697$

A1: $x_3=awrt\ 0.4732$,and $x_4=awrt 0.4728$

Ignore subscripts, mark in the order given.

(c)

M1: Choose suitable interval for x, e.g. $[0.4725,\ 0.4735]$ attempts f(x) at each.

If they use a different function it must be defined or implied by sight of the expression.

For example, candidates could attempt $\pm g(x)$ at each where $g(x) = \left(x - \frac{1}{2}e^{-\frac{x^2}{4}} \right)$.

FYI $g(0.4725) = -0.00036$ $g(0.4735) = +0.00075$

A minority of candidate may choose a tighter range which should include 0.47282 (alpha to 5dp),
This would be acceptable for both marks, provided the conditions for the A mark are met.
Continued iteration is M0

A1: needs (i) both evaluations correct to 1 sf, (either rounded or truncated) or 3 dp

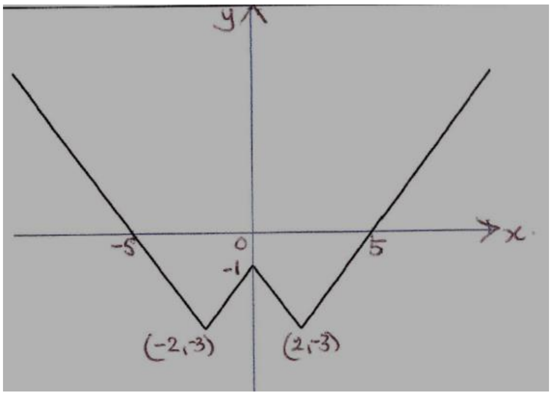
(ii) sign change stated (or implied by $f(a) \times f(b) < 0$) oe and

(iii)some form of conclusion which may be :

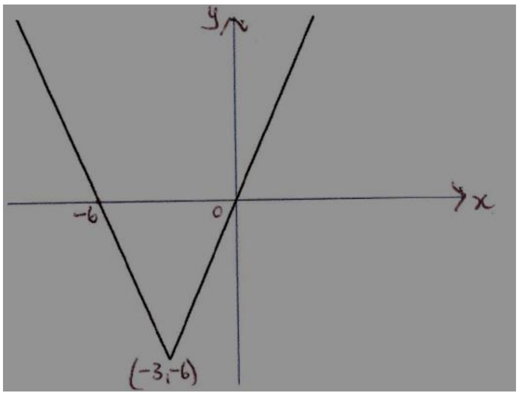
$\Rightarrow root = 0.473$ or "so result shown" or qed or tick or equivalent

x	f(x)
0.4725	-0.000756289
0.4726	-0.000521044
0.4727	-0.000285838
0.4728	-5.06723E-05
0.4729	0.000184454
0.473	0.00041954
0.4731	0.000654587
0.4732	0.000889594
0.4733	0.001124561
0.4734	0.001359489
0.4735	0.001594377

x	g(x)
0.4725	-0.000357482
0.4726	-0.000246309
0.4727	-0.000135135
0.4728	-2.39585E-05
0.4729	8.72201E-05
0.473	0.000198401
0.4731	0.000309584
0.4732	0.000420769
0.4733	0.000531956
0.4734	0.000643145
0.4735	0.000754336

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4(a)</td><td>

M1: W shape anywhere, (so allow this mark for $y = |f(x)|$). Condone a lack of symmetry
A1: Intercepts at (5, 0), $(-5,0)$ and $(0, -1)$. Allow 5, -5 and -1 written on the correct axes. Do NOT allow (0,5) for (5,0) etc.
A1: vertices (corresponding to P) at both $(2, -3)$ and $(-2, -3)$</td><td>M1 A1 A1

(3)</td></tr>
<tr><td>(b)</td><td>

M1: V shape (correct way up) anywhere on the page. Do not however award for the original V. Condone a lack of symmetry or it appearing as a tick.
A1: intercepts through O and $(-6, 0)$ Allow -6 written on the correct axis. Do NOT allow (0, -6) for (-6, 0). Remember the M must have been scored.
A1: Single vertex at $(-3, -6)$</td><td>M1 A1 A1

(3)
(6 marks)</td></tr>
</table>

Question Number	Scheme	Marks
5(a)	$\dfrac{9(4+x)}{16-9x^2} \equiv \dfrac{A}{(4-3x)} + \dfrac{B}{(4+3x)} \Rightarrow A \text{ or } B$	M1
	$A = 6$ or $B = 3$ obtained at any point of the solution	A1
	$\dfrac{9(4+x)}{16-9x^2} \equiv \dfrac{6}{(4-3x)} + \dfrac{3}{(4+3x)}$	A1
		(3)
(b)	$\displaystyle\int \dfrac{9(4+x)}{16-9x^2}\,dx \equiv \int \dfrac{A}{(4-3x)} + \dfrac{B}{(4+3x)}\,dx$	
	$= -\dfrac{A}{3}\ln(4-3x) + \dfrac{B}{3}\ln(4+3x)\ (+c)$	M1 A1ft
	$(= -2\ln(4-3x) + \ln(4+3x)\ (+c))$	
	$= \ln\dfrac{(4+3x)}{(4-3x)^2} + c\ , = \ln\dfrac{k(4+3x)}{(4-3x)^2}$ or $\ln\left\|\dfrac{k(4+3x)}{(4-3x)^2}\right\|$	M1, A1
		(4)
		(7 marks)

This question may be marked as one

(a)

M1: Sets or implies $\dfrac{9(4+x)}{16-9x^2} \equiv \dfrac{A}{(4-3x)} + \dfrac{B}{(4+3x)}$ and proceeds to find at least one unknown

Sets or implies $\dfrac{(4+x)}{16/9 - x^2} \equiv \dfrac{A}{(4/3 - x)} + \dfrac{B}{(4/3 + x)}$ and proceeds to find at least one unknown

Sets or implies $\dfrac{9(4+x)}{16-9x^2} \equiv \dfrac{A}{(-3x-4)} + \dfrac{B}{(3x-4)}$ and proceeds to find at least one unknown

Condone $\dfrac{9(4+x)}{16-9x^2} \equiv \dfrac{A}{(3x-4)} + \dfrac{B}{(3x+4)}$ and proceeds to find at least one unknown

A1: Either constant correct or one correct fraction

A1: $\dfrac{9(4+x)}{16-9x^2} \equiv \dfrac{6}{(4-3x)} + \dfrac{3}{(4+3x)}$ in either (a) or within the integral in (b)

Alternative correct forms are;

$\dfrac{2}{(4/3 - x)} + \dfrac{1}{(4/3 + x)}$, $\quad -\dfrac{6}{(3x-4)} + \dfrac{3}{(3x+4)}$,

$\dfrac{-3}{(-3x-4)} + \dfrac{-6}{(3x-4)} \quad \dfrac{1.5}{(1-\frac{3}{4}x)} + \dfrac{0.75}{(1+\frac{3}{4}x)}$

Watch out for $\dfrac{9(4+x)}{16-9x^2} \equiv \dfrac{6}{(3x-4)} - \dfrac{3}{(3x+4)}$ where we see 6 and -3 but scores M1 A0 A0

(b)

M1: Uses their partial fractions from part (a) and integrates to obtain $...\ln(4-3x)+...\ln(4+3x)$ or equivalent such as $...\ln\left(\frac{4}{3}-x\right)+...\ln\left(\frac{4}{3}+x\right)$ with or without modulus signs.

If they fail to reach $\frac{9(4+x)}{16-9x^2}\equiv\frac{A}{(4-3x)}+\frac{B}{(4+3x)}$ or an alternative correct form and use say $\frac{9(4+x)}{16-9x^2}\equiv\frac{A}{(x)}+\frac{B}{(16-9x)}$ candidate can (potentially) score the first three marks in part (b) as long as they have two fractions.

A1ft: Correct answer for their *A*, *B* (do not need constant of integration at this stage) – may have modulus signs

M1: For combining their log terms correctly with a constant of integration seen on the same line.

A1: cao. The answer given in the scheme o.e.

Allow $-2\ln(4-3x)+\ln(4+3x)+c\to\ln\frac{k(4+3x)}{(4-3x)^2}$ without explanation or

$-2\ln(4-3x)+\ln(4+3x)+c\to\ln\frac{e^c(4+3x)}{(4-3x)^2}$ or

$\frac{-3}{(-3x-4)}+\frac{-6}{(3x-4)}+c\to\ln\left|\frac{k(-3x-4)}{(3x-4)^2}\right|$ **with the modulus sign**

N.B. $\ln\frac{(4+3x)}{(4-3x)^2}+c$ gets M1 A0 as does $-2\ln(4-3x)+\ln(4+3x)+c\to\ln\frac{c(4+3x)}{(4-3x)^2}$

Question Number	Scheme	Marks
6	$(V)=\pi\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\tan\left(\frac{x}{2}\right)\mathrm{d}x$	B1
	$=(\pi)\left[-6\ln\cos\left(\frac{x}{2}\right)\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $(\pi)\left[6\ln\sec\left(\frac{x}{2}\right)\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$	M1A1
	$=(\pi)\left[-6\ln\left(\frac{1}{\sqrt{2}}\right)+6\ln\left(\frac{\sqrt{3}}{2}\right)\right]$	dM1
	$=(\pi)\left[6\ln\left(\frac{\sqrt{6}}{2}\right)\right]=3\pi\ln\left(\frac{3}{2}\right)$	A1 **(5)**
		(5 marks)

B1: Need the expression including π and correct limits. The limits and π may be implied by later working. Condone the omission of the $\mathrm{d}x$. You do not need to see V

As a minimum accept $\pi\int 3\tan\frac{x}{2}$ with the limits $\frac{\pi}{2}$ and $\frac{\pi}{3}$ being used later

or $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\tan\frac{x}{2}$ found and subsequently multiplied by π

M1: Achieves $k\ln\cos(x/2)$ or $k\ln\sec(x/2)$ where k is constant

A1: cao – do not need π nor limits. It is for $-6\ln\cos\left(\frac{x}{2}\right)$ or $6\ln\sec\left(\frac{x}{2}\right)$ oe

Note that it may be common to see a first line of $(V)=2\pi\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\tan\left(\frac{x}{2}\right)\mathrm{d}x$.

In this case you would award for $-12\ln\cos\left(\frac{x}{2}\right)$ or $12\ln\sec\left(\frac{x}{2}\right)$

dM1: Dependent on first M1. Substitutes given limits and subtracts (either way around)

A1: cao and depends on having explicitly seen evidence for both M marks

Question Number	Scheme	Marks
7 (a)	$\begin{pmatrix}13\\15\\-8\end{pmatrix}+\lambda\begin{pmatrix}3\\3\\-4\end{pmatrix}=\begin{pmatrix}7\\-6\\14\end{pmatrix}+\mu\begin{pmatrix}2\\-3\\2\end{pmatrix}\Rightarrow \begin{matrix}13+3\lambda=7+2\mu\\15+3\lambda=-6-3\mu\\-8-4\lambda=14+2\mu\end{matrix}$ **any two** of these	M1
	Full method to find either λ **or** μ	M1
	$(1)-(2)\Rightarrow\mu=-3$	
	Sub $\mu=-3$ into (2) to give $\lambda=-4$ (need both*)	A1
	Check values in 3rd equation $-8-4\times-4=14-6=8$ (True)	B1
	Position vector of intersection is $\begin{pmatrix}13\\15\\-8\end{pmatrix}+-4\begin{pmatrix}3\\3\\-4\end{pmatrix}$ OR $\begin{pmatrix}7\\-6\\14\end{pmatrix}+-3\begin{pmatrix}2\\-3\\2\end{pmatrix}=$	dM1
	$=\begin{pmatrix}1\\3\\8\end{pmatrix}$	A1 **(6)**
(b)	$\cos\theta=\dfrac{\begin{pmatrix}3\\3\\-4\end{pmatrix}.\begin{pmatrix}2\\-3\\2\end{pmatrix}}{\sqrt{3^2+3^2+(-4)^2}\sqrt{2^2+(-3)^2+2^2}}=\dfrac{-11}{17\sqrt{2}}$	M1 A1
	So acute angle is awrt 62.8 degrees or awrt 1.10 radians	A1 **(3)**
(c)	When $\lambda=-6$ this gives $\begin{pmatrix}-5\\-3\\16\end{pmatrix}$ so A lies on l_1	B1 **(1)**
(d)	Vector approach: $\overrightarrow{AB}=6\mathbf{i}+6\mathbf{j}-8\mathbf{k}$ or "Bus stop" approach: At C $\lambda=-2$	M1
	$\overrightarrow{BC}=6\mathbf{i}+6\mathbf{j}-8\mathbf{k}$ and **c** = **b** + $\overrightarrow{BC}$ oe, or so $\begin{pmatrix}13\\15\\-8\end{pmatrix}+-2\begin{pmatrix}3\\3\\-4\end{pmatrix}$	M1
	$\overrightarrow{OC}=7\mathbf{i}+9\mathbf{j}$	A1 **(3)**
		(13 marks)

(a)

M1: For writing down **any two equations** that give the coordinates of the point of intersection.

Accept two of $13+3\lambda=7+2\mu$, $15+3\lambda=-6-3\mu$, $-8-4\lambda=14+2\mu$

There must be an attempt to set the coordinates equal but condone one slip in total in the two equations.

M1: A full method to find **either** λ **or** μ.

A1: Both values correct $\mu=-3$ and $\lambda=-4$ (need both). Correct values following correct equations implies M1 A1

NB * It is possible to provide a complete proof by solving two of the three equations to give $\mu = -3$ or $\lambda = -4$ and then to solve the third equation with one of the previous equations to give the same value independently. It is then sufficient for this mark to have just one of $\mu = -3$ or $\lambda = -4$.

B1: The correct values must be substituted into **both** sides of the third equation. There must be some minimal statement (a tick will suffice) that the values are the same. This can also be scored via the substitution of $\mu = -3$ $\lambda = -4$ into **both** of the equations of the lines but there must be the same minimal statement.

For example $8 = 8$ is insufficient evidence but $8 = 8$ ✓ is fine

NB* It is possible to provide a complete proof by solving two of the three equations to give $\mu = -3$ or $\lambda = -4$ and then to solve the third equation with one of the previous equations to give the same value independently. It is then sufficient for this mark to have just one of $\mu = -3$ or $\lambda = -4$ but there must be the same minimal statement that the lines meet.

dM1: Substitutes their value of λ into l_1 to find the coordinates or position vector of the point of intersection. It is dependent upon having scored second method mark. Alternatively substitutes their value of μ into l_2 to find the coordinates or position vector of the point of intersection.

A1: Correct answer only. Accept as a vector or a coordinate. Accept (1, 3, 8) (A correct answer here implies previous M mark)

Note that it is possible to score 1,1,1,0,1,1

(b)

M1: A clear attempt to use the correct formula for $\cos\theta =$ using the scalar product of the direction vectors. Allow for one slip and proceed to $\cos\theta =$ a fraction or decimal.

If they attempt to use the point of intersection and another point on each line they **must** get a multiple of the direction vectors.

A1: For $\frac{\pm 11}{17\sqrt{2}}$ or equivalent - may be implied by 62.8 or 117.2 or 1.10 radians or 2.04 radians

A1: cao for awrt 62.8 or 1.10 radians

(c)

B1: Shows that $\lambda = -6$ in all three cases **and** draws conclusion – e.g. point lies on line, or result shown, or QED, or tick….

Alternatively substitutes $\lambda = -6$ in $\begin{pmatrix} 13+3\times -6 \\ 15+3\times -6 \\ -8-4\times -6 \end{pmatrix} = \begin{pmatrix} -5 \\ -3 \\ 16 \end{pmatrix}$ and gives a (minimal) conclusion Eg ☐

(d)

M1: A correct attempt at any correct vector in the direction of AB or BA using $\overrightarrow{OA}$ and their $\overrightarrow{OB}$.
Allow if two components are correct.

For example $\overrightarrow{AB} = 6\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}$ or $\overrightarrow{BC} = 6\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}$ or $\overrightarrow{AC} = 12\mathbf{i} + 12\mathbf{j} - 16\mathbf{k}$

If the bus stop approach is used it is for attempting to find $\lambda(=-2)$ at C

M1: A fully correct method to find $\overrightarrow{OC}$ Using a vector approach $\mathbf{c} = \mathbf{b}$ + their $\overrightarrow{AB}$ or $\mathbf{c} = \mathbf{a} + 2\times$ their $\overrightarrow{AB}$ or $\mathbf{c} = -\mathbf{a} + 2\times\mathbf{b}$ Other methods are possible.

A1: $\overrightarrow{OC} = 7\mathbf{i} + 9\mathbf{j}$ or $\overrightarrow{OC} = \begin{pmatrix} 7 \\ 9 \\ 0 \end{pmatrix}$ Do NOT accept just the coordinate $(7, 9, 0)$

The correct vector without working scores 111, the correct coordinates 110

question Number	Scheme	Marks
8	Differentiates wrt x $\quad \frac{dy}{dx} = 16\sec^2(2x)$ oe	M1
	Inverts to get $\frac{dx}{dy} = \frac{1}{16\sec^2 2x} = \frac{1}{16(1+\tan^2 2x)}$ $\quad$ | $\quad \frac{dy}{dx} = 16(1+\tan^2(2x)) = 16\left(1+\left(\frac{y}{8}\right)^2\right)$	dM1
	$\frac{dx}{dy} = \frac{A}{B+y^2}$	ddM1
	$= \frac{4}{64+y^2}$	A1
		(4)
		(4 marks)

M1: Achieves $\frac{dy}{dx} = \lambda\sec^2(2x)$ oe or implicitly $1 = \lambda\sec^2(2x)\frac{dx}{dy}$

If they change $\tan 2x$ to $\frac{\sin 2x}{\cos 2x}$ they can score this mark for $\frac{dy}{dx} = \frac{\alpha\cos 2x\cos 2x \pm \beta\sin 2x\sin 2x}{(\cos 2x)^2}$

If they change $\tan 2x$ to $\frac{2\tan x}{1-\tan^2 x}$ they could never reach the required solution so score M0

dM1: Scored for two of the three processes 1 and 2 (either order) or 2 followed by 3 :

1. The reciprocal must be taken. (The variable cannot change)
2. The identity $1+\tan^2 2x = \sec^2 2x$ must be attempted
3. There must be an attempt to replace $\tan 2x$ by $\frac{y}{8}$

ddM1: Scored for attempting all three processes **and** attempting to eliminate the fractions (seen in at least two of the terms in the expression)

A1: cso

Alternative using arctan

M1: Expresses x as $x = \lambda\arctan\left(\frac{y}{8}\right)$ and attempts some differentiation $\Rightarrow \frac{dx}{dy} = \frac{...}{...+...y^2}$

dM1: As above but achieves $\frac{dx}{dy} = \frac{C}{(1+\left(\frac{y}{8}\right)^2)}$

ddM1: Eliminates fractions (seen in at least two of the terms in the expression) $\frac{dx}{dy} = \frac{A}{B+y^2}$

A1: cso $\frac{dx}{dy} = \frac{4}{64+y^2}$

Question Number	Scheme	Marks
9 (a)	**Way 1 LHS → RHS:** $\frac{\cot^2 x}{1+\cot^2 x} \equiv \frac{\cos^2 x / \sin^2 x}{\text{cosec}^2 x}, \equiv \frac{\cos^2 x / \sin^2 x}{1/\sin^2 x} \equiv \cos^2 x$	M1 M1, A1*
	Way 2 LHS → RHS:: $\frac{\cot^2 x}{1+\cot^2 x} \times \frac{\sin^2 x}{\sin^2 x} \equiv \frac{\cos^2 x}{\sin^2 x + \cos^2 x}, \equiv \frac{\cos^2 x}{1}, \equiv \cos^2 x$	M1, M1, A1*
	Way 3 LHS → RHS: $\frac{\cot^2 x}{1+\cot^2 x} \equiv \frac{1/\tan^2 x}{1+1/\tan^2 x} \equiv \frac{1}{1+\tan^2 x} \equiv \frac{1}{\sec^2 x}, \equiv \cos^2 x$	M1, M1, A1*
	Way 4: LHS → RHS:: $\frac{\cot^2 x}{1+\cot^2 x} \equiv \frac{\text{cosec}^2 x - 1}{\text{cosec}^2 x} \equiv 1-\sin^2 x \equiv \cos^2 x$	M1 M1 A1*
	Way 5: Considers both sides	
	$\cos^2 x(1+\cot^2 x) = \cos^2 x\left(1+\frac{\cos^2 x}{\sin^2 x}\right) = \frac{\cos^2 x(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x} = \cot^2 x$	M1 M1
	Hence $\frac{\cot^2 x}{1+\cot^2 x} \equiv \cos^2 x$	A1*
		(3)
(b)	$\cos^2 x = 8(2\cos^2 x - 1) + 2\cos x$	M1
	$15\cos^2 x + 2\cos x - 8 = 0$	A1
	So $\cos x = 2/3$ or $-4/5$	M1
	$\Rightarrow x = 48.2°$ or $143.1°$ or $216.9°$ or $311.8°$	dM1, A1 A1
		(6)
		(9 marks)

(a)

M1: Uses one VALID identity may implied. It is usually one of

$1+\cot^2 x = \text{cosec}^2 x$, $1+\tan^2 x = \sec^2 x$, $\cot^2 x = \cos^2 x / \sin^2 x$, $\cot^2 x = 1/\tan^2 x$ $\text{cosec}^2 x = 1/\sin^2 x$,

$1/\text{cosec}^2 x = \sin^2 x$, $1-\sin^2 x \equiv \cos^2 x$, $\cot^2 x \sin^2 x = \cos^2 x$, $\sec^2 x = 1/\cos^2 x$, $1/\sec^2 x = \cos^2 x$

$\cot^2 x \sin^2 x = \cos^2 x$

M1: Uses two VALID identities (not the same one twice) may implied.

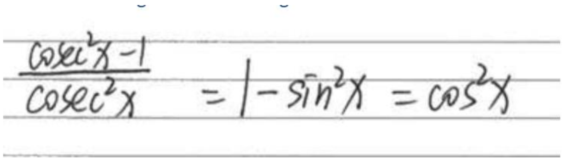
A1: See a complete process.

All notation must be correct (~~$\cos x^2$~~ **) including correct use of variables** ~~$\text{cosec}^2 = \frac{1}{\sin^2}$~~

However, condone a lack of variables if it does not form part of their proof (and is an aside)

There will be some combinations of these methods. **A complete method with no errors scores** M1M1A1

Way 5 must contain a conclusion for the A mark.



For example: The above minimum response can be marked as follows

M1: A correct identity $1+\cot^2 x = \text{cosec}^2 x$ used

M1: A second identity $\frac{1}{\text{cosec}^2 x} = \sin^2 x$ implied. Alternatively, could be scored for $1-\sin^2 x \equiv \cos^2 x$ used

A1: Completes proof with no errors and correct notation.

(b)

M1: Attempt to use both part (a) to replace left hand side and the correct double angle formula $\cos 2x = 2\cos^2 x - 1$ on right hand side to form an equation in $\cos x$ only. If, for instance, $\cos 2x = \cos^2 x - \sin^2 x$ is used this mark is not scored until the $\sin^2 x$ has been replaced by $1-\cos^2 x$

Condone a slip or an omission on either of the coefficients 8 and 2.

For example, $\cos^2 x = 2\cos^2 x - 1 + 2\cos x$ or $\cos^2 x = 8(2\cos^2 x - 1) + \cos x$ if fine for M1

A1: Correct three term quadratic with all terms on same side of equation. The = 0 may be implied by subsequent work

M1: Solves quadratic in $\cos x$ by any method – factorising, formula or completion of square or just writing down answers. Correct answers imply this M mark.

It is dependent upon having attempted to replace $\cos 2x$ by $\pm 2\cos^2 x \pm 1$ oe

dM1: For proceeding to find one correct answer for **their** inverse cos. You may have to use a calculator.

It is dependent upon the previous M mark. One correct answer implies this mark

A1: Two correct answers from awrt $48.2°, 143.1°, 216.9°, 311.8°$

or awrt 0.841, 5.442, 2.498 and 3.785 which are the radian solutions.

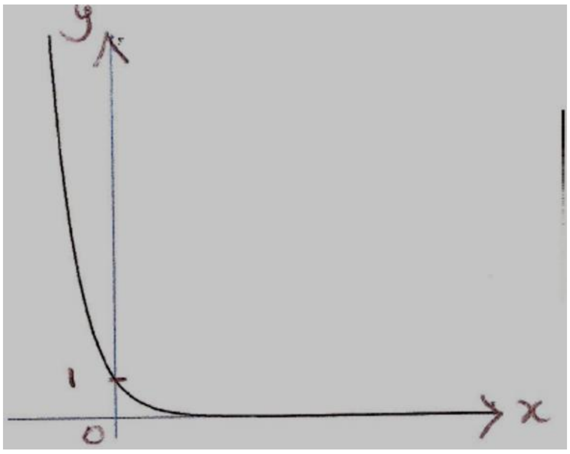
These cannot fortuitously be awarded from incorrect working.

A1: All four answers in the range, $x = awrt\ 48.2°$, $143.1°$, $216.9°$ and $311.8°$ and no others

Any extra solutions in the range withhold the final A mark.

Ignore any solutions outside the range $0 \leqslant x \leqslant 360°$

Due to the complexities in this question we will not be applying the misread rule for students who miscopy the equation in (b)

Question Number	Scheme	Marks
10(a)	 Correct shape or intercept at 1 Fully correct	M1 A1 **(2)**
(b)	g(x) > 1	M1 A1 **(2)**
(c)	$y = \dfrac{x}{x-3} \quad \Rightarrow (x-3)y = x \quad \Rightarrow xy - x = 3y$ $\Rightarrow x = \dfrac{3y}{y-1}$ $g^{-1}(x) = \dfrac{3x}{x-1}$, with $x >$ "1"	M1 dM1 A1, B1ft **(4)**
(d)	Way 1: $e^{-2\left(\frac{x}{x-3}\right)} = 3$ | Way 2: $g\left(x\right) = f^{-1}(3)$ $-2\left(\dfrac{x}{x-3}\right) = \ln 3$ | $g(x) = -\frac{1}{2}\ln 3$ $-2x = \ln 3(x-3)$ so $x =$ | $x = g^{-1}\left(-\frac{1}{2}\ln 3\right)$ $x = \dfrac{3\ln 3}{2 + \ln 3}$	M1 A1 dM1 A1 **(4)** **(12 marks)**

(a)

M1: For a correct shape (any position) or a curve passing through 1 on the +ve y - axis .
For the curve, look for a gradient that is always negative and increasing. Condone slips of the pen.
For the y intercept condone it being marked (1,0) but do not accept e^0

A1: Correct shape with y - intercept at 1 and asymptotic to the x - axis.
As a rule of thumb look for it reaching a point that is half way below the intercept at 1 with some levelling out. Do not condone the y intercept marked as (1,0) for this mark.

(b)

M1: Finds the value 1 but incorrect inequality is possible.

For example $y \geqslant 1, x > 1, \mathrm{f}(x) > 1,\ \mathrm{g} \neq 1,\ \ 1 < y < 7$ or even g(x) < 1

A1: Needs g(x) > 1, g > 1, y > 1, $(1, \infty)$ or $(1; \infty)$ but do not accept $\mathrm{f}(x) > 1$

(c)

M1: Setting $y =$ multiplying across and attempting to collect x terms. Award for $\pm xy \pm x = \pm 3y$ but condone numerical slips. Alternatively starting with $x = \dfrac{y}{y-3}$ multiples across and attempts to collect y terms.

If it is attempted by division then expect to see $y = \dfrac{x}{x-3} \Rightarrow y = A + \dfrac{B}{x-3}$ before the A is moved across

dM1: Dependent upon the previous M mark. It is for an attempt at making x or a replaced y the subject of formula. Look for $x = \dfrac{\pm 3y}{\pm y \pm 1}$ but condone numerical slips.

A1: For $\mathrm{g}^{-1}(x) = \dfrac{3x}{x-1}$ or exact equivalent such as $\mathrm{g}^{-1}(x) = -\dfrac{3x}{1-x}$ or $\mathrm{g}^{-1}(x) = 3 + \dfrac{3}{x-1}$ or $\mathrm{g}^{-1}(x) = 3 - \dfrac{3}{1-x}$

Do not allow $\mathrm{y} = \dfrac{3x}{x-1}$ or $\mathrm{f}^{-1}(x) = \dfrac{3x}{x-1}$

B1ft: domain x > 1 or ft their range from part (b) as long as it is in x or set form $(1, \infty)$ Condone $(1; \infty)$

Don't follow through on $y \in \mathbb{R}$ following $x \in \mathbb{R}$

(d)

M1: Way 1 for an attempt at setting fg(x) =3 Condone slips but the order of operations must be correct.
Way 2 for using $\mathrm{g}(x) = \mathrm{f}^{-1}(3)$

A1: Undoes the exponentials to reaches a correct equation in x.

So either $-2\left(\dfrac{x}{x-3}\right) = \ln 3$ or $\dfrac{x}{x-3} = -\tfrac{1}{2}\ln 3$ or $\mathrm{g}(x) = -\tfrac{1}{2}\ln 3$

dM1: A full attempt to make x the subject of the formula from two x terms. It is dependent upon the previous M.
Apply the same rules for change of subject as for M1 dM1 in (c)
Alternatively attempts $\mathrm{g}^{-1}\mathrm{f}^{-1}(3)$ following through on their g^{-1}

A1: $x = \dfrac{3\ln 3}{2 + \ln 3}$ or exact equivalent e.g. ln3 may appear as − ln(1/3) or − 1/2 (ln9)

Question Number	Scheme		Marks
11 (a)	(3,0) and (-3, 0)		B1, B1 **(2)**
(b)	$\frac{\pi}{2}$ and $\frac{3\pi}{2}$		M1 A1 **(2)**
(c)	$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{18\cos 2t}{-3\sin t}$		M1 A1
	$=\frac{18\times\frac{1}{2}}{-3\times\frac{1}{2}}=-6$		dM1 A1 **(4)**
(d)	$y^2=81\times4\sin^2 t\cos^2 t$	$y=9\times2\sin t\cos t$	M1
	Attempts to replace $\cos^2 t=\frac{x^2}{9}$ *and* $\sin^2 t=1-\frac{x^2}{9}$	Attempts to replace $\cos t=\frac{x}{3}$ *and* $\sin t=\sqrt{1-\frac{x^2}{9}}$	M1
	Correct eqn $y^2=81\times4\times\left(1-\frac{x^2}{9}\right)\times\frac{x^2}{9}$	Correct eqn $y=9\times2\times\sqrt{1-\frac{x^2}{9}}\times\frac{x}{3}$	A1
	Obtain $y^2=4x^2(9-x^2)$	Obtain $y^2=4x^2(9-x^2)$	A1 **(4)**
			(12 marks)

(a)

B1: For one correct value of x or for seeing $(0,3)$ or $(0,-3)$

B1: For both **coordinates** correct. You can ignore any reference to A or B or O

(b)

M1: For one correct in degrees or radians. You may well see students who work out both $\cos t=0$ and $\sin 2t=0$ and produce many values of t without selecting the correct ones or even selecting incorrect ones. They can have access to the M mark

A1: For both correct in radians (and no others inside the range)

(c)

M1: Attempts to differentiate both x and y wrt t and uses $\frac{dy/dt}{dx/dt}$

You may see candidates who attempt to set $y=18\sin t\cos t$ before differentiating.

Condone poor/ incorrect differentiation for the method.

A1: Correct result with no errors seen. $\frac{dy}{dx}=\frac{18\cos 2t}{-3\sin t}$ or exact equivalent, for example $\frac{dy}{dx}=\frac{18\cos^2 t-18\sin^2 t}{-3\sin t}$

(NB if ${}^{dy}\!/_{dt}$ and ${}^{dx}\!/_{dt}$ have the 'wrong' sign – this is A0)

dM1: Attempts to substitute $t = \frac{\pi}{6}$ into their trig expression for $\frac{dy}{dx} =$

It is dependent upon having scored the previous M1.

A1: cso

Generally

M1: Attempts to use the double angle formula for $\sin 2t$ to reach $y = 18\sin t\cos t$ or equivalent.

You may see this after squaring y so $y^2 = 81\times 4\sin^2 t\cos^2 t$ Condone $y^2 = 162\sin^2 t\cos^2 t$

M1: Uses correct trig identities to form an equation linking y with x

This usually involves using both $x = 3\cos t$ *and* $\sin t = \sqrt{1-\cos^2 t}$

Condone for this mark $x = \cos t$

A1: A correct intermediate equation

A1: cso $y^2 = 4x^2(9-x^2)$

Alt method 1: Uses both sides and given result

M1: Substitutes $x = 3\cos t$ and $y = 9\sin 2t$ into $y^2 = ax^2\left(b-x^2\right)$ and attempts the double angle formula for $\sin 2t$ Eg $81\times 4\sin^2 t\cos^2 t = a9\cos^2 t\left(b-9\cos^2 t\right)$

M1 : Proceeds so that both sides are of the same form and attempts to find at least one unknown

Eg Replace $\sin^2 t$ by $1-\cos^2 t$ on lhs $\Rightarrow 81\times 4\left(1-\cos^2 t\right)\cos^2 t = a9\cos^2 t\left(b-9\cos^2 t\right)$ multiplies out $\Rightarrow 324\cos^2 t - 324\cos^4 t = 9ab\cos^2 t - 81a\cos^4 t$, and then solves two equations of the form $..a = ..$ **and** $...ab = ...$ to find one unknown.

A1: Solves two equations of the form $..a = ..$ **and** $...ab = ...$ to find both unknowns with one value correct.

A1: Correct equation $y^2 = 4x^2(9-x^2)$ or $a = 4, b = 9$ **and** states hence true

Note that it is possible to find $b = 9$ by substituting $(\pm 3, 0)$ into $y^2 = ax^2\left(b-x^2\right)$. This scores no marks

Alt method 2: Uses $\sin^2 2t = 1-\cos^2 2t$ both sides and given result

M1: Attempts to square and use $\sin^2 2t = 1-\cos^2 2t$ Eg $y^2 = k\sin^2 2t = k\left(1-\cos^2 2t\right)$

M1: Attempts to use $\cos 2t = 2\cos^2 t - 1$ and $x = 3\cos t$ to form an equation linking y with x

Condone for this mark $x = \cos t$

A1: In this method it could be $y^2 = 81\left(1-\left(1-\frac{2x^2}{9}\right)^2\right)$

A1: cso $y^2 = 4x^2(9-x^2)$

Question Number	Scheme	Marks
12. (a)	$R=\sqrt{4+16}=\sqrt{20}$ *or* $2\sqrt{5}$	B1
	$\tan\alpha=\frac{4}{2}$	M1
	$\Rightarrow\ \alpha=1.11$ (awrt)	A1
		(3)
(b)	Maximum is 12+2*R* or minimum is 12– 2*R*	M1
	maximum = 20.9 (hours) (20h 57m) and minimum = 3.06 (hours) (3 hours 3 m)	A1 A1
		(3)
(c)	$17=12+k\,"R"\sin\left(\frac{2\pi t}{365}\pm"\alpha"\right)$	M1
	$\sin\left(\frac{2\pi t}{365}\pm"\alpha"\right)=....$	dM1
	For proceeding to one value for *t* from $17=12+2"R"\sin\left(\frac{2\pi t}{365}\pm"\alpha"\right)$	M1
	t = 99 (days) **or** 212 or 213 (days)	A1
	For finding two values for t	dM1
	t = 99 (days) **and** 212 or 213 (days)	A1
		(6)
		(12 marks)

(a)

B1: $R=\sqrt{20}$ or $2\sqrt{5}$ no working needed. Condone $R=\pm\sqrt{20}$ oe

M1: $\tan\alpha=\pm\frac{4}{2}$ or $\tan\alpha=\pm\frac{2}{4}$ and attempts to find alpha. If *R* is used accept $\sin\alpha=\pm\frac{4}{"R"}$ or $\cos\alpha=\pm\frac{2}{"R"}$

A1: accept $\alpha=$ awrt 1.11 ; also accept $\sqrt{20}\sin(x-1.11)$. Answers in degrees are A0

(b)

M1: Uses Maximum is 12+2*R* or minimum is 12– 2*R* with their value of *R*

A1: maximum value or minimum value correct allowing exact value(s) $12\pm2\sqrt{20}$ or $12\pm4\sqrt{5}$

A1: maximum **and** minimum value **awrt** 20.9 (20h 57m) 3.06 (3 hours 3 m)
Ignore any units in this part.

Note: It is possible to do this by differentiation. To score M1 you would need to see Differentiation to $\lambda\cos\left(\frac{2\pi t}{365}-'\alpha'\right)=0\Rightarrow\frac{2\pi t}{365}-'\alpha'=\frac{\pi}{2}$ or $\frac{3\pi}{2}\Rightarrow t=...$ and then substitute into *H* and find a value.

(c)

M1: For an attempt to interpret the model and writing it in terms of (a), condoning slips

Allow for $17 = 12 + k\text{"}R\text{"}\sin\left(\frac{2\pi t}{365} \pm \text{"}\alpha\text{"}\right)$, even $k = 1$ with their value for R and α (Slip on "2")

Allow $17 = 12 + k\text{"}R\text{"}\sin\left(x \pm \text{"}\alpha\text{"}\right)$ even $k = 1$ with their value for R and α (x instead of $\frac{2\pi t}{365}$)

dM1: For attempting to make $\sin\left(x \pm \text{their } \alpha\right)$ or $\sin\left(\frac{2\pi t}{365} \pm \text{"}\alpha\text{"}\right)$ the subject.

M1: For the method of finding at least one value for t, $0 < t < 365$, from a "**correct**" starting point with $2 \times$ their R.

$17 = 12 + 2\text{"}R\text{"}\sin\left(\frac{2\pi t}{365} \pm \text{"}\alpha\text{"}\right) \rightarrow \sin\left(\frac{2\pi t}{365} \pm \text{"}\alpha\text{"}\right) = C$ to t = ... by undoing the operations in the correct order

A good intermediate value to check (for correct R) is $\frac{2\pi t}{365} \pm \text{"}\alpha\text{"} = 0.593....$

Condone slips on the $\frac{2\pi}{365}$ for all M marks. Example you may see $\frac{2\pi}{36}$

A1: For one correct value for t, either awrt 99 **or** awrt 212/213.

dM1: For attempting to find a second value for t.

It is dependent upon the previous M mark and it is usually for moving from

$\left(\frac{2\pi t}{365} \pm \text{"}\alpha\text{"}\right) = \pi - \beta$ (where β was the principal value) to t = ...

by undoing the operations in the correct order

A good intermediate value to check (for correct R) is $\frac{2\pi t}{365} \pm \text{"}\alpha\text{"} = 2.548....$

A1: awrt 99 **and** awrt 212 or 213 only $0 < t < 365$. Remember to ISW

Question Number	Scheme	Marks
13 (a)	$\frac{1}{2}\times \mathrm{e}\times\{\ldots\ldots\ldots\}$	B1 oe
	$\frac{1}{2}\times h\times\{0.3114+0.1215+2(0.2195+0.1712+0.1416)\}$	M1
	$=2.04$ (3 sf)	A1
		(3)
(b)	Let $u=\ln 2x$ then $\frac{du}{dx}=\frac{2}{2x}$	B1
	So $\int \frac{1}{2x}\ln 2x\mathrm{d}x=\int \frac{1}{2}u\mathrm{d}u=\frac{1}{4}[\ln(2x)]^2$ oe	M1 A1
		(3)
(c)	$\left[\frac{1}{4}(\ln 2x)^2\right]_{\mathrm{e}}^{5\mathrm{e}} = \left[\frac{1}{4}(\ln 10\mathrm{e})^2-\frac{1}{4}(\ln 2\mathrm{e})^2\right]=2.01$	M1 A1
		(2)
(d)	Way 1: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2x}\times\frac{2}{2x}-\frac{1}{2x^2}\ln 2x$ $=\left(\frac{1}{2x^2}-\frac{1}{2x^2}\ln 2x\right)$ | Way 2: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x\times\frac{2}{2x}-(\ln 2x)\times 2}{(2x)^2}$ $=\left(\frac{1-(\ln 2x)}{2x^2}\right)$	M1 A1
	When $x=\frac{e^2}{2}$, $y=\frac{2}{\mathrm{e}^2}$	B1
	Uses $\left(\frac{\mathrm{e}^2}{2},"\frac{2}{\mathrm{e}^2}"\right)$ with their $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\rvert_{\frac{\mathrm{e}^2}{2}}$ to form equation of the tangent	dM1
	$y=-\frac{2}{\mathrm{e}^4}x+\frac{3}{\mathrm{e}^2}$ cao	A1
		(5)
		(13 marks)

(a)

B1: See $\frac{1}{2}\times \mathrm{e}\times$ as part of trapezium rule or $h=\mathrm{e}$ stated or used

M1: Correct structure of the terms inside the {.....} of the trapezium rule with their *h*. Expect to see $\frac{1}{2}\times h\times\{0.3114+0.1215+2(0.2195+0.1712+0.1416)\}$ condoning slips on the digits of the numbers.

Award this mark if the bracket { } is not present $\frac{1}{2}\times h\times 0.3114+0.1215+2(0.2195+0.1712+0.1416)$

A1: awrt 2.04 Condone $\frac{599}{800}\mathrm{e}$ or awrt $0.749\mathrm{e}$

(b) Mark parts (b) and (c) together
Hence

B1: Finds $\frac{du}{dx}=\frac{2}{2x}$ or exact equivalent

M1: Integrates as far as ku^2 or $k[\ln(2x)]^2$ or equivalent

A1: cao $=\frac{1}{4}\left[\ln(2x)\right]^2 \quad =\frac{1}{4}\left[\ln 2x\right]^2 \quad =\frac{1}{4}\ln^2(2x) \quad \text{or} =\frac{1}{4}\ln^2 2x$ FINAL ANSWER

May be awarded in (c) (Does not need constant of integration)

Note $=\frac{1}{4}\ln(2x)^2$ or $=\frac{1}{4}\ln 2x^2$ is incorrect

(b) Otherwise

B1 M1: Integrates as far as $k\left[\ln(2x)\right]^2$

A1: cao $=\frac{1}{4}\left[\ln(2x)\right]^2 \quad =\frac{1}{4}\left[\ln 2x\right]^2 \quad =\frac{1}{4}\ln^2(2x) \quad \text{or } =\frac{1}{4}\ln^2 2x$ FINAL ANSWER

May be awarded in (c) (Does not need constant of integration)

Note $=\frac{1}{4}\ln(2x)^2$ or $=\frac{1}{4}\ln 2x^2$ is incorrect

(c)

M1: Uses correct limits correct way round in an integrated function.
Condone a poor attempt at integrating but the limits cannot be substituted into the original function.
If the integral is left in terms of *u* then the limits must be $\ln 2\text{e}=1.69...$ to $\ln 10\text{e} =3.30...$

A1: Correct answer. Accept awrt 2.01 Allow recovery from incorrect notation $=\frac{1}{4}\ln(2x)^2$,

(d)

M1: Attempts to differentiate $y=\frac{1}{2x}\ln 2x$ using either the product rule or quotient rule to achieve either

$\frac{\text{d}y}{\text{d}x}=\frac{A}{x^2}-\frac{B\ln 2x}{x^2}$ $(A,B>0)$ for the product rule

$\frac{\text{d}y}{\text{d}x}=\frac{\frac{Px}{x}-Q(\ln 2x)}{(2x)^2}$ oe with $(P,Q>0)$ for the quotient rule.

Condone $2x^2$ for $(2x)^2$ on the denominator.

A1: Correct use of quotient or product rule– may not be simplified – accept any correct answer.

B1: Correct simplified *y* coordinate $\frac{2}{\text{e}^2}$ oe such as 2e^{-2} may be awarded within a tangent (or normal equation)

dM1: Uses their $\left(\frac{\text{e}^2}{2},\text{"}\frac{2}{\text{e}^2}\text{"}\right)$ with their $\left.\frac{\text{d}y}{\text{d}x}\right|_{x=\frac{\text{e}^2}{2}}=\frac{A}{x^2}-\frac{B\ln 2x}{x^2}$ (or equivalent) to form the equation of the tangent. It is dependent upon the M mark for differentiation. Accept $y-\text{"}\frac{2}{\text{e}^2}\text{"}=\text{"}-\frac{2}{\text{e}^4}\text{"}\left(x-\frac{\text{e}^2}{2}\right)$

If the form $y=mx+c$ is used it is for proceeding as far as $c=..$

A1 cao. It must be simplified. If the simplified *y* coordinate was not written down but this is correct and simplified, you should retrospectively award the B1

Question Number	Scheme	Marks
14. (a)	$\frac{dV}{dr} = 4\pi r^2$	B1 **(1)**
(b)	$\frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr} = \frac{9000\pi}{(t+81)^{\frac{5}{4}}} \times \frac{1}{4\pi r^2}$	M1
	$= \frac{2250}{r^2(t+81)^{\frac{5}{4}}}$	A1 **(2)**
(c)	$\frac{dr}{dt} = \frac{k}{r^n(t+81)^{\frac{5}{4}}}$, so $\int r^n \, dr = \int \frac{k}{(t+81)^{\frac{5}{4}}} dt$	B1
	$\frac{r^3}{3} = \frac{2250}{-1/4} \times (t+81)^{-\frac{1}{4}} (+c)$	M1 A1ft
	When $t = 0$, $r = 3$ so $c = 9 + 9000 \times (+81)^{-\frac{1}{4}}$	M1
	So $\frac{r^3}{3} = -9000 \times (t+81)^{-\frac{1}{4}} + 3009$	dM1
	$r = \left[9027 - 27000(t+81)^{-\frac{1}{4}} \right]^{\frac{1}{3}}$	A1 **(6)**
(d)	Uses $t = 175$ to give $r = 13.2$	B1 **(1)**
(e)	Uses $t = 175$ to give $\frac{dr}{dt} = 0.0127$ or 0.0126	M1 A1 **(2)**
		(12 marks)

(a)

B1: cao. Condone $= \frac{4}{3} \times 3\pi r^2$ You may isw after a correct answer

(b)

M1: Correct use of chain rule $\frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr}$ with $\frac{dV}{dt}$ (allow slips) and their $\frac{dV}{dr}$

Allow any correct version $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with $\frac{dV}{dt}$ (allow slips) and their $\frac{dV}{dr}$ leading to $\frac{dr}{dt} =$

A1: cao $\left(\frac{dr}{dt} = \right) \frac{2250}{r^2(t+81)^{\frac{5}{4}}}$ Condone $r \leftrightarrow R, t \leftrightarrow T$

Watch for fudging here. The correct $\frac{dr}{dt}$ can easily appear to be found by incorrect chain rule. This is M0 A0

If the chain rule is written down it must be correct. There needs to be an attempt to use it for the M mark.
For A1 there must not be any incorrect lines.

(c)

B1: Correct separation of variables for their $\frac{\mathrm{d}r}{\mathrm{d}t}$. (Now a B mark) Allow with or without the integral signs but the $\mathrm{d}r$ and $\mathrm{d}t$ must be present and on the correct side as numerators.

M1: Correct method of integration on both sides, following through on n only

Look for $\int r^n\,\mathrm{d}r = \int \frac{k}{(t+81)^{\frac{5}{4}}}\,\mathrm{d}t \to ...r^{n+1} = ...(t+81)^{-\frac{1}{4}}$ with or without + c

A1ft: Correct integration of both sides ft their k and n – need not have c

M1: The equation must now have a constant. It is for using t = 0, r = 3 in order to find c. Condone poor attempts at integration for this method mark.

dM1: This is dependent upon having achieved $...\frac{r^3}{} = ...(t+81)^{-\frac{1}{4}} +$ numerical c

It is for proceeding to r = using a correct order of operations

Alternatively score this mark for correctly achieving $r^3 = 9027 - 27000(t+81)^{-\frac{1}{4}}$ (SC)

A1: $r = \left[9027 - 27000(t+81)^{-\frac{1}{4}}\right]^{\frac{1}{3}}$

(d)

B1: r = 13.2 (accept awrt 13.2)

Note: We can retrospectively award the dM1 in (c), for an answer of 13.2 in (d).

(e)

M1: Substitutes $t = 175$ and their r = 13.2 into $\frac{\mathrm{d}r}{\mathrm{d}t} =$

You may need to use a calculator here. It may be implied by 1sf rounded or truncated

A1: $\frac{\mathrm{d}r}{\mathrm{d}t} = 0.0126$ or 0.0127 (awrt)

……………………………………………………………………………………………………..

Special case 1: Where evidence in (d) can be used in (c)

There may be students who do part (c) without ever achieving **the** formula for r in terms of t

Eg answer to (c) is $r = \sqrt[3]{C - 27000 \times (t+81)^{-\frac{1}{4}}}$

Then in part (d) they use $t = 0, r = 3$ and go on to find $r = ..,$ when $t = 175$

If they state $C = 9027$ and go on to correct give $r = 13.2$ they can score all marks in (c)

If they don't find $C = 9027$ but use say

$\left[\frac{r^3}{3}\right]_3^R = \left[-9000 \times (t+81)^{-\frac{1}{4}}\right]_0^{175} \Rightarrow r = 13.2$ just withhold the final A1 in (c)

……………………………………………………………………………………………………..

Special case 2: Candidates who miscopy 9000 as 900 and achieve the following would lose the A mark in (b) and the final A1 in (c). Rules for a misread/miscopy 12-2=10 marks maximum

(b) $= \frac{225}{r^2(t+81)^{\frac{5}{4}}}$ (c) $r = \left[927 - 2700(t+81)^{-\frac{1}{4}}\right]^{\frac{1}{3}}$ (d) r = 6.32 (e) $\frac{\mathrm{d}r}{\mathrm{d}t} = 0.00551$

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