Mark Scheme (Results)

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Pearson Edexcel International GCSE in Further Pure Mathematics (4PM0)
Paper 02

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**
  Always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  Any case of suspected misread loses 2A (or B) marks on that part, but can gain the M marks.
  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

Follow through marks

Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.

- **Linear equations**
  Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right), \text{ where } |pq|=|c| \quad \text{leading to } x= \ldots.$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \quad \text{leading to } x= \ldots.$$

2. Formula:
Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x= \ldots.$
3. Completing the square:

$$x^2+bx+c=0: \ \left(x\pm\tfrac{b}{2}\right)^2 \pm q \pm c=0, \quad q\neq 0 \qquad \text{leading to } x= \ldots.$$

Method marks for differentiation and integration:

1. Differentiation
Power of at least one term decreased by 1. $\left(x^n \to x^{n-1}\right)$
2. Integration:
Power of at least one term increased by 1. $\left(x^n \to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:
The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required.
(Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question Number	Scheme	Marks
1 (a)	$192 = \frac{\theta}{2} \times 12^2 \Rightarrow \theta = \frac{8}{3}$ (radians) accept 2.67 or better	M1A1 (2)
(b)	$l = 12 \times \frac{8}{3} = 32$ (cm)	M1A1 (2)
ALT	Use $A = \frac{1}{2} rl$ $192 = 6l \Rightarrow l = 32$ M1A1	[4]
(a)		
M1	Use of correct formula. If the formula for use with degrees is used, must change to radians for the answer to gain this mark	
A1	Correct answer	
(b)		
M1	Use of either correct (radian) formula with their angle from (a) or the degree formula	
A1	Correct answer awrt 32	

Question Number	Scheme	Marks
2(a)	$\sum_{r=1}^{n}(3r+2)=\frac{n}{2}(2\times5+(n-1)3)=\frac{n}{2}(7+3n)$ *	M1A1cso (2)
ALT	Splitting terms:	
	$\sum_{r=1}^{n}(3r+2)=\sum_{1}^{n}3r+\sum_{1}^{n}2=3\times\frac{n(n+1)}{2}+2n,=\frac{n}{2}(7+3n)$ *	M1,A1 (2)
(b)	$\sum_{r=10}^{20}(3r+2)=\sum_{r=1}^{20}(3r+2)-\sum_{r=1}^{9}(3r+2)$	M1
	$=\frac{20}{2}(7+3\times20)-\frac{9}{2}(7+3\times9)=517$	A1A1 (3)
ALT	$\sum_{r=10}^{20}(3r+2)=\frac{11}{2}(32+62)=517$ M1A1A1	
		[5]
(a)		
M1	Use $S=\frac{n}{2}(2a+(n-1)d)$ or $\frac{n}{2}(a+l)$ showing the correct substitution	
A1cso	Reach **given** result with no errors seen	
ALT:		
M1	Split the sum into 2 parts and use either sum formula on $\sum_{r=1}^{n}3r$ or use the standard result for the sum of the first n natural numbers. Allow if $\sum_{r=1}^{n}2$ or $2n$ seen	
A1cso	Reach **given** result with no errors seen	
(b)		
M1	Use the difference of two sums with upper limit 9 or 10 for second sum	
A1	Substitute correct numbers (n = 9 now)	
A1cao	517	
ALT		
M1	Use summation formula with $n=11$ or $10, a=32, l=62$	
A1	Substitute correct numbers (n = 11 now)	
A1cao	517	
	NB: (b) can be done by listing the terms and adding them 32 + 35 + ...+ 62 with an answer seen is minimum for M1 Ignore any intermediate terms if shown. A2 correct answer Correct answer with no working shown scores 0/3	

Question Number	Scheme	Marks
3	$x=1,\ \ y^2=4 \Rightarrow y=\pm 2$	B1
	Volume = $\pi\int_{-2}^{2}\left(5-y^2\right)\,\mathrm{d}y-\pi\int_{-2}^{2}1\,\mathrm{d}y, =\left(\pi\right)\left(\left[5y-\dfrac{y^3}{3}\right]_{-2}^{2}-\left[y\right]_{-2}^{2}\right)$	M1,M1
	$=\left(\pi\right)\left\{\left(10-\dfrac{8}{3}\right)-\left(-10-\dfrac{-8}{3}\right)-\left(2--2\right)\right\}=\dfrac{32\pi}{3}$ (units^3)	dM1A1cao
ALT	B1 Limits as above	
	Volume = $\pi\int_{-2}^{2}\left(5-y^2\right)\,\mathrm{d}y-\pi\times 1\times 4=\pi\left[5y-\dfrac{y^3}{3}\right]_{-2}^{2}-4\pi$ M1M1	
	$=\pi\left\{\left(10-\dfrac{8}{3}\right)-\left(-10-\dfrac{-8}{3}\right)\right\}-4\pi=\dfrac{32\pi}{3}$ (units^3) M1A1	
		[5]
	Notes cover either method	
B1	Correct y coords for points of intersection (shown explicitly or only seen as limits)	
M1	Use $\pi\int x^2\mathrm{d}y$ for volume of curve, with an attempt to obtain an integrand in terms of y, and cylinder by integral or standard volume formula. This mark can only be awarded when evidence of a **difference** of these volumes is seen. Limits not needed.	
M1	Attempt the integration of their dimensionally correct curve integral. (ie not squared). Integration must be wrt y. Limits and π not needed. **Algebraic integration must be seen.**	
dM1	Substitute their limits in their integrated function and obtain a value for the volume of the cylinder using consistent values. π not needed. Depends on 2nd M mark (but not first)	
A1cao	Correct volume (as shown or equivalent multiple of π eg 10.7 π)	
	NB: All marks are available if work is done without π but π included in the final answer.	

Question Number	Scheme	Marks
4(a)	$b^2-4ac>0 \quad p^2-4\times3\times4>0$	M1
	$\Rightarrow p^2>48 \Rightarrow$ critical values are $p=\pm\sqrt{48} \quad \left(=\pm4\sqrt{3}\right)$	dM1A1
	So set of values; $p<-4\sqrt{3},\, p>4\sqrt{3}$ (accept 3dp or better inc $\pm\sqrt{48}$)	ddM1A1 (5)
(b)	$\pm6,\pm5,\pm4,\pm3,\pm2,\pm1,0$	B1 (1) [6]
(a)	For the first **3** marks accept an equation or any inequality sign. For the first **4** marks accept the use of x instead of p	
M1	Use discriminant	
dM1	Solve to find the CVs Depends on the first M mark.	
A1	Correct CVs, exact or (min) 3 dp	
dM1	Form 2 inequalities for the **outside** regions using their CVs ie p < smaller CV and p > larger CV Depends on both previous M marks	
A1	Correct set of values	
(b)		
B1	Answer as shown.	

Question Number	Scheme	Marks
5	$\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{x}\left(3x^{2}-6\right)+12x\mathrm{e}^{x}$	M1A1A1
	$\frac{\mathrm{d}^{2}y}{\mathrm{d}x^{2}}=\left[2\mathrm{e}^{x}\left(3x^{2}-6\right)+12x\mathrm{e}^{x}\right]+\left[12x\mathrm{e}^{x}+12\mathrm{e}^{x}\right]$	M1A1
	$\frac{\mathrm{d}^{2}y}{\mathrm{d}x^{2}}=\mathrm{e}^{x}\left(6x^{2}+2x\right)$	
	$\frac{\mathrm{d}^{2}y}{\mathrm{d}x^{2}}-2\frac{\mathrm{d}y}{\mathrm{d}x}+y=12\mathrm{e}^{x}$ *	dM1A1 (7) [7]
ALT	First derivative as above	M1A1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=y+12x\mathrm{e}^{x}$	
	$\frac{\mathrm{d}^{2}y}{\mathrm{d}x^{2}}=\frac{\mathrm{d}y}{\mathrm{d}x}+12x\mathrm{e}^{x}+12\mathrm{e}^{x}$	M1A1
	Substitute both derivatives in the given result	dM1
	Correct answer as given, no errors seen	A1

M1	Differentiate using the product rule. The sum of 2 terms reqd.
A1	Either term correct. No simplification needed. Ignore errors made when simplifying.
A1	Both terms correct. No simplification needed. Ignore errors made when simplifying.
M1	Differentiate again using the product rule *correctly* on at least one term. (The term the product rule is applied to need not be correct.)
A1	Any correct result for $\frac{\mathrm{d}^{2}y}{\mathrm{d}x^{2}}$ (ie their second derivative should be equivalent to the one shown)
dM1	Complete, probably by substituting their derivatives in LHS of the given result. At least one intermediate step (eg full substitution or bracketed equation above must be seen) Depends on both previous M marks.
A1	Fully correct final result reached with no errors seen.

Question Number	Scheme	Marks
6(a)	$\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$	B1 (1)
(b)	$\overrightarrow{PQ} = \frac{\mathbf{a}}{4} + \frac{1}{2}(-\mathbf{a} + \mathbf{b}),\ = \frac{1}{4}(2\mathbf{b} - \mathbf{a})$ oe	M1,A1 (2)
(c)(i)	$\overrightarrow{QR} = \frac{\mu}{4}(2\mathbf{b} - \mathbf{a})$	B1 ft
(ii)	$\overrightarrow{QR} = \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \lambda\mathbf{b}$	M1A1 (3)
(d)(i)	$\frac{2\mu}{4}\mathbf{b} - \frac{\mu}{4}\mathbf{a} = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a} + \lambda\mathbf{b} \Rightarrow -\frac{\mu}{4} = -\frac{1}{2} \Rightarrow \mu = 2$	M1M1A1
(ii)	$\frac{2\mu}{4} = \frac{1}{2} + \lambda \Rightarrow \lambda = \frac{1}{2}$	A1 (4)
		[10]
(a)B1	Correct vector	
(b)M1	$\overrightarrow{PQ} = \frac{\mathbf{a}}{4} + \frac{1}{2}\left(\text{their } \overrightarrow{AB}\right)$ or $\overrightarrow{PO} + \overrightarrow{OB} + \overrightarrow{BQ} = -\frac{3}{4}\mathbf{a} + \mathbf{b} - \frac{1}{2}\left(\text{their } \overrightarrow{AB}\right)$	
A1	Correct vector $\overrightarrow{PQ}$	
(c)		
(i)B1ft	$\frac{\mu}{4}\left(\text{their } \overrightarrow{PQ}\right)$	
(ii)M1	$\overrightarrow{QR} = \frac{1}{2}\left(\text{their } \overrightarrow{AB}\right) + \lambda\mathbf{b}$	
A1	Correct vector as shown or with **b** terms collected	
(d)		
M1	Equate their 2 forms for $\overrightarrow{QR}$	
M1	Equate coeffs of **a** and obtain a value for μ or equate coefficients of **a** and **b** and get a value of λ Both versions of $\overrightarrow{QR}$ must have an **a** term and a **b** term	
(i)A1	Correct value of μ (or λ)	
(ii)A1	Complete to obtain the correct value of the second unknown.	

Question Number	Scheme	Marks
7(i)	$\dfrac{\left(8^x\right)^x}{32^x}=4\Rightarrow\dfrac{2^{3x^2}}{2^{5x}}=2^2\Rightarrow 2^{\left(3x^2-5x\right)}=2^2$	M1A1
	$\Rightarrow 3x^2-5x=2\Rightarrow(3x+1)(x-2)=0$	M1
	$x=-\dfrac{1}{3},2$	A1 (4)
(ii)	$\log_x 64-\log_x 4=\log_x\left(\dfrac{64}{4}\right)=\log_x 16$	M1
	$\log_x 16=\dfrac{\log_4 16}{\log_4 x}=\dfrac{2}{\log_4 x}$	M1
	$3\log_4 x+\dfrac{2}{\log_4 x}=5\Rightarrow 3(\log_4 x)^2+2=5\log_4 x$	M1
	$\Rightarrow 3(\log_4 x)^2-5\log_4 x+2=0$	
	$\Rightarrow(3\log_4 x-2)(\log_4 x-1)=0$	dM1
	$\Rightarrow\log_4 x=\dfrac{2}{3},\log_4 x=1$	A1
	$\Rightarrow x=4^{\frac{2}{3}}\left(=2^{\frac{4}{3}}=\sqrt[3]{16}\right)=2.5198421...\approx 2.52$ or better $\quad x=4^1=4$	dM1A1 (7)
		[11]
ALT (ii)	$\log_x 64-\log_x 4=\log_x\left(\dfrac{64}{4}\right)=\log_x 16=2\log_x 4$	M1
	$\log_4 x=\dfrac{\log_x x}{\log_x 4}=\dfrac{1}{\log_x 4}$	M1
	$2\log_x 4+\dfrac{3}{\log_x 4}=5\Rightarrow 2(\log_x 4)^2+2=5\log_x 4$	M1
	$\Rightarrow 2(\log_x 4)^2-5\log_x 4+3=0$	
	$\Rightarrow(2\log_x 4-3)(\log_x 4-1)=0$	dM1
	$\Rightarrow\log_x 4=\dfrac{3}{2},\log_x 4=1\Rightarrow 4=x^{\frac{3}{2}},4=x^1$	A1
	$\Rightarrow x=4^{\frac{2}{3}}=\left(\sqrt[3]{16}\right)=2.5198421...\approx 2.52$ or better, $\quad x=4^1=4$	dM1A1 (7)

Question Number	Scheme	Marks
(i)		
M1	Change all terms of equation to powers of 2 (or possibly 4)	
A1	Correct 2 term equation with powers of 2	
M1	Equate the powers in their equation and solve the resulting 3 term quadratic	
A1	Correct values for x (both needed)	
	Special Case: Using factor theorem:	
	Substitute $x=2$ and show correct M1A1M0A0	
	If (unlikely) same done with $x=-\frac{1}{3}$ - send to Review!	
(ii)		
	The work for the first 3 M marks may appear in a different order.	
	Enter the marks in the order shown here.	
M1	Combine the two logs base x or combine the equivalent logs after changing base. Award for combining the 2 equivalent numbers after multiplying through by their denominators	
M1	Change all logs base x to logs base 4 (or all logs to the same base)	
M1	Obtain a 3 term quadratic, terms in any order.	
dM1	Solve their 3 term quadratic to $\log_4 x = ...$ or $\log_p x = ...$ Depends on all previous M marks	
A1	Two correct values for $\log_4 x$ or $\log_p x$	
dM1	"Undo" their logs to get at least one value for x (not nec correct) Depends on all previous M marks.	
A1	Two correct values for x. Accept accurate answers or min 3 sf	
ALT		
M1	Combine the two logs base x	
M1	Change all log base 4 to log base x	
M1	Obtain a 3 term quadratic, terms in any order.	
dM1	Solve their 3 term quadratic to $\log_x 4 = ...$ or $\log_x p = ...$ Depends on all previous M marks	
A1	Two correct values for $\log_x 4$ or $\log_x p$	
dM1	"Undo" their logs to get at least one value for x Depends on all previous M marks	
A1	Two correct values for x	

Question Number	Scheme	Marks
8 (a)	$355=\pi r^2 h \Rightarrow h=\dfrac{355}{\pi r^2}$ or $\pi rh=\dfrac{355}{r}$	B1
	$S=2\pi r^2+2\pi rh \Rightarrow S=2\pi r^2+2\pi r\left(\dfrac{355}{\pi r^2}\right)=2\pi r^2+\dfrac{710}{r}$ *	M1A1 A1cso (4)
(b)	$\dfrac{\mathrm{d}S}{\mathrm{d}r}=4\pi r-710r^{-2}$	M1
	$4\pi r-710r^{-2}=0 \Rightarrow 4\pi r=\dfrac{710}{r^2} \Rightarrow r^3=\dfrac{710}{4\pi}$	dM1
	$r=\sqrt[3]{\dfrac{710}{4\pi}}$ $(r=3.837215...)$ cm	A1
	$S=2\pi\times\left(\sqrt[3]{\dfrac{710}{4\pi}}\right)^2+\dfrac{710}{\sqrt[3]{\dfrac{710}{4\pi}}}=277.5450...\approx 278\ (\text{cm}^2)$	dM1A1cao (5)
(c)	$\dfrac{\mathrm{d}^2S}{\mathrm{d}r^2}=4\pi+\dfrac{1420}{r^3}$ $\left\{=4\pi+\dfrac{1420}{3.837215^3}=37.699\right\}$	M1
	(r positive so) $\dfrac{\mathrm{d}^2S}{\mathrm{d}r^2}>0$ $\therefore S$ is minimum	A1ft (2)
		[11]

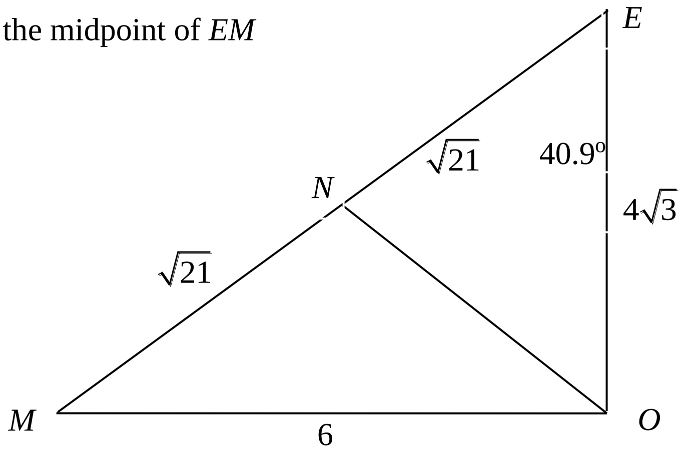
(a)	
B1	$h=\dfrac{355}{\pi r^2}$ or $\pi rh=\dfrac{355}{r}$ ..seen explicitly
M1	Use a correct formula for the surface area and substitute their expression for h which must have been seen explicitly. (eg $S=2\pi r^2+2\pi rh \Rightarrow S=2\pi r^2+\left(\dfrac{355\times 2}{r}\right)$ alone scores M0 as does any other re-arrangement of the answer.)
A1	Correct expression for h used
A1cso	Obtain the **given** result from a fully correct solution. Must see S = ...
(b)	
M1	Differentiate the **given** expression for S - power of either term to decrease
dM1	Equate their derivative to 0 and solve for r Depends on the first M mark
A1	Correct value for r, exact or decimal (3 sf sufficient) seen explicitly or used to calculate the minimum value of S.
dM1	Substitute their value for r in the given expression for S Depends on the previous 2 M marks.
A1cao	278
(c)	
M1	Obtain the second derivative (or use an other method to test for a min value of S). Methods involving testing value of S on either side of their value of r or looking at the change of sign of the first derivative must include evaluating S or dS/dt
A1ft	Concluding (correct) statement. No need to evaluate the second derivative provided their value of r is positive and the second derivative is algebraically correct. (Ignore incorrect evaluation unless negative.)

Question Number	Scheme	Marks
9(a)	$\dfrac{x^3-2x^2-5x+6}{x+2}=x^2-4x+3$	M1
	$x^2-4x+3=(x-3)(x-1)\Rightarrow x=1\ ,\ 3$	M1(NB A1 on e-PEN)
(i)	so $a=1$ *	A1cso
(ii)	$b=3$	A1 (B1 on e-PEN) (4)
	Correct answers w/o working scores 0/4	
(b)	$\dfrac{dy}{dx}=3x^2-4x-5$ when $x=2$, $\dfrac{dy}{dx}=-1$	M1A1
	$x=2,\ \ y=-4$	B1
	$(y--4)=-1(x-2)$	M1A1
	when $y=0, x=-2$ *	A1cso (6)
ALT		
	$x=2,\ \ y=-4$	B1
	$\dfrac{dy}{dx}=3x^2-4x-5$ when $x=2$, $\dfrac{dy}{dx}=-1$	M1A1
	Grad of line from P to $(-2,0)=-1$	M1A1
	Same gradient so l passes through (-2,0)	A1cso
(c)	Area $=\int_{-2}^{2}(x^3-2x^2-5x+6)\ dx-\int_{-2}^{2}(-x-2)\ dx=\int_{-2}^{2}(x^3-2x^2-4x+8)\ dx$	M1
	$=\left[\dfrac{x^4}{4}-\dfrac{2x^3}{3}-2x^2+8x\right]_{-2}^{2}$	M1
	$=\left(4-\dfrac{16}{3}-8+16\right)-\left(4+\dfrac{16}{3}-8-16\right)$	dM1
	$=\dfrac{64}{3}$	A1 (4)
		[14]
ALT	By splitting the area:	
	Area $=\int_{-2}^{1}(x^3-2x^2-5x+6)\ dx+\Delta[(-2,0),(2,0),P]-\left\|\int_{1}^{2}(x^3-2x^2-5x+6)\ dx\right\|$	M1
	Integrate curve equation (ignore limits) and attempt area of triangle by formula or integration	M1
	Substitute **correct** limits	dM1
	$=\dfrac{64}{3}$	A1
NB	No algebraic integration – only first M mark available	

Question Number	Scheme	Marks
(a) **(i)M1** **M1** **(A1 on e-PEN)** **A1cso** **(ii)A1** **(B1 on e-PEN)**	Obtain the quadratic factor by division or inspection Factor theorem allowed **only** if values for *a* **and** *b* are found. Factorise the quadratic factor Correct **given** value for *a* Correct value for *b* By Factor Theorem: M1 Test $x=1$ M1 Test another value which is > 1 A1 $a=1$ A1 $b=3$	
(b) **M1** **A1** **B1** **M1** **A1** **A1cso** **ALT**	Differentiate and substitute *x* = 2 to find the gradient of the tangent to *C* at *P* Correct gradient of tangent $y=-4$ seen explicitly or used in the equation of the tangent Any **complete** method for the equation of the tangent at $(2,\text{ their } y)$. Use of $y=mx+c$ must include an attempt at finding a value for *c* Correct numbers in their (unsimplified) equation Correct *x* coordinate of the point where the tangent crosses the *x*-axis. No errors seen **for the last 3 marks:** Find the gradient of the line from *P* to $(-2,0)$ M1 Any correct method; A1correct gradient All work correct and a conclusion. A1cso	
(c) **M1** **M1** **dM1** **A1** **ALT** **M1** **M1** **dM1** **A1**	Using area $=\int \text{curve}-\text{line}$ or $\int \text{line}-\text{curve}$ with their line equation limits **are** needed Attempt to integrate the single function or two functions (ie all the integration needed) limits not needed. $\int_{-2}^{2}(-x-2)\,\mathrm{d}x$ may be obtained by triangle formula. Substitute **correct** limits in their integrated function(s) Depends on both M marks Correct final answer. Must be positive. By splitting the area Suitable split eg as shown Limits are needed Attempt all the nec integration (ignore limits) and area of triangle by integration or formula Substitute **correct** limits in their integrated function(s) Depends on both M marks Correct final answer	

Question Number	Scheme	Marks
10(a)	$\frac{y--4}{-4-1}=\frac{x--6}{-6-4}\Rightarrow y+4=\frac{1}{2}(x+6)$ oe eg $y=\frac{1}{2}x-1$	M1A1 (2)
(b)	$\left(\frac{3\times4+2\times-6}{5},\frac{3\times1+2\times-4}{5}\right)\Rightarrow(0,-1)$	M1A1 (2)
(c)	Gradient of perpendicular = −2	B1
	Allow all following work if *x*, *y* used instead of *m*, *n*	
	$-2=\frac{n--1}{m-0}\quad(\Rightarrow -2m=n+1)$	B1ft
	$\left(3\sqrt{5}\right)^2=(m-0)^2+(n--1)^2\Rightarrow 45=m^2+(n+1)^2$	
	$45=m^2+4m^2\Rightarrow 45=5m^2\Rightarrow m=\pm3$ negative required $m=-3$	M1A1
	$\Rightarrow n=-2m-1\Rightarrow n=-2\times-3-1=5$ coordinates are $(-3,\ 5)$	A1 (5)
(d)(i)	$RQ=\sqrt{(-13--3)^2+(0-5)^2}=5\sqrt{5}$	M1
	$AB=\sqrt{(4--6)^2+(1--4)^2}=5\sqrt{5}$	
	With conclusion	A1cso
(ii)	$\left(\text{Gradient of } AB=\frac{1}{2}\right)$ Gradient of $RQ=\frac{5-0}{-3--13}=\frac{1}{2}$	M1
	With conclusion *	A1cso (4)
ALT	By vectors – combines both parts:	
	$\overrightarrow{AB}=10\mathbf{i}+5\mathbf{j}$ or equivalent column vector	M1
	$\overrightarrow{RQ}=10\mathbf{i}+5\mathbf{j}$ or equivalent column vector	M1
	So same length and parallel (provided both vectors are correct)	A1A1
(e)	Area is base × height $A=3\sqrt{5}\times5\sqrt{5}=75$ (units)^2	M1A1 (2) [15]
ALT:	$A=\frac{1}{2}\begin{vmatrix}-3 & 4 & -6 & -13 & -3\\ 5 & 1 & -4 & 0 & 5\end{vmatrix}$ M1	
	$=\frac{1}{2}\left[(-3-20)+(-16+6)+(0-52)-(65-0)\right]=-75\Rightarrow 75$ A1	

Question Number	Scheme	Marks
(a)		
M1	Any complete method for obtaining an equation of l	
A1	Correct equation in any form inc unsimplified	
(b)		
M1	Obtaining at least one of the coords of P. Must be correct. Can be by formula or diagram.	
A1	Both coords correct.	
	NB: If both coords are just written down, award M1A1 if both correct; M0A0 otherwise	
(c)		
B1	Correct gradient of the perpendicular	
B1ft	Correct equation connecting m and n from equating their gradient to -2 Can be unsimplified. Follow through their gradient of the perpendicular but must be negative reciprocal of gradient of l	
M1	Use Pythagoras (with + sign as shown oe)to find the length of PQ, equate this to $3\sqrt{5}$ and solve to $m = ...$	
A1	Correct value for m ± 3 allowed here	
A1	Correct value for n Values do not have to be written in coordinate brackets. Only **one** final answer or this mark is lost.	
(d)		
(i)M1	Use Pythagoras to find the length of RQ or AB	
A1cso	Lengths of both lines correct with working for each and a conclusion shown	
(ii)M1	Find the gradient of RQ Must show working	
A1cso	Correct gradient of both lines and a conclusion shown	
ALT	M1M1 one M mark for each vector correct or working shown but slip made A1A1 one A mark for each conclusion **provided** the vectors are correct.	
(e)		
M1	Obtaining the area of $ABPQ$ by using the formula for the area of a parallelogram	
A1	Correct area	
ALT:	Use the "determinant" method.	
M1	Formula must be correct ie $\frac{1}{2}$ needed, 5 pairs of coordinates with first and last the same, coordinates to be in order round the quadrilateral (clockwise or anticlockwise). An attempt to evaluate also needed.	
A1	Correct area - must be positive.	

Question Number	Scheme	Marks
11(a)	$AC=\sqrt{12^2+8^2}\ \left(=\sqrt{208}=4\sqrt{13}\right)$ or AO or $OC=\sqrt{6^2+4^2}\ \left(=2\sqrt{13}\right)$	M1
	$h=\sqrt{10^2-52}=\sqrt{48}=4\sqrt{3}$ *	M1A1cso (3)
(b)	$\angle OCE=\cos^{-1}\left(\frac{2\sqrt{13}}{10}\right)=43.8537...\approx 43.9^{\circ}$	M1A1 (2)
	or $\sin^{-1}\left(\frac{4\sqrt{3}}{10}\right)$ or $\tan^{-1}\left(\frac{4\sqrt{3}}{2\sqrt{13}}\right)$	
(c)	Let M be the midpoint of BC.	
	$EM=\sqrt{10^2-4^2}=2\sqrt{21}$	M1
	$\cos\theta^{\circ}=\frac{4\sqrt{3}}{2\sqrt{21}}=\frac{2\sqrt{7}}{7}$ *	M1A1cso (3)
	or cosine rule $\cos\theta^{\circ}=\frac{\left(4\sqrt{3}\right)^2+\left(2\sqrt{21}\right)^2-6^2}{2\times4\sqrt{3}\times2\sqrt{21}}=\frac{2\sqrt{7}}{7}$	
(d)	Let N be the midpoint of EM 	
	$NO=\sqrt{\left(\sqrt{21}\right)^2+\left(4\sqrt{3}\right)^2-2\times\sqrt{21}\times4\sqrt{3}\times\frac{2\sqrt{7}}{7}}..=\sqrt{21}$	M1A1ftA1
	hence triangle NEO is isoceles, so required angle ($\angle ENO$)	
	$\angle ENO=180-2\times40.8933...=98.2134...\approx98.2^{\circ}$	B1 (4) [12]
ALT	Based on symmetry:	
	$\tan\frac{\theta}{2}=\frac{\left(\frac{h}{2}\right)}{3}=\frac{2\sqrt{3}}{3}$	M1A1
	$\frac{\theta}{2}=49.1066...$	A1
	$\theta=98.2°$	A1(B1 on e-PEN)

Question Number	Scheme	Marks
(a)		
M1	Use Pythagoras with a + sign to find *AC* or *AO* or*OC*	
M1	Use Pythagoras with a $-$ sign to find the height	
A1cso	**Given** height obtained from correct working. No decimals used but allow $2\sqrt{13}=7.2...$ followed by $7.2...^2=52$	
(b)		
M1	Use any trig function to obtain angle *OCE*	
A1	Correct size of angle *OCE* Must be 1 dp	
(c)		
M1	Use Pythagoras with a $-$ sign to obtain the length of *EM* (need not be correct)	
M1	$\cos\theta^{\circ}=\dfrac{4\sqrt{3}}{EM}$ with their *EM* or cosine rule as shown. Must reach $\cos\theta=...$ if other form used at start. (NB not dependent)	
A1cso	Correct completion to the **given** answer	
(d)		
M1	Use of cosine rule in ΔEON to obtain ON	
A1ft	Correct numbers follow through their *EM*	
A1	Correct length *ON*, exact or awrt 4.58	
B1	**Correct** size of angle, must be 1 dp unless already penalised in (b). (Can be obtained by the isos triangle as shown or by cosine or sine rule in ΔEON)	
	NB: No A1ft in alt method as *h* is given in (a)	