Mark Scheme (Results)

January 2018

Pearson Edexcel International GCSE in
Further Pure Mathematics (4PM0)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

  Types of mark
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

  Abbreviations
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  Any case of suspected misread loses A (and B) marks on that part, but can gain the M marks.
  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.
  If there is a choice of methods shown, then no marks should be awarded, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer on the answer line then check the working for an obvious answer.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right),\text{ where } |pq|=|c| \text{ leading to } x=....$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)\text{ where } |pq|=|c| \text{ and } |mn|=|a| \text{ leading to } x=....$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$$x^2+bx+c=0:\ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0 \qquad \text{leading to } x=....$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\rightarrow x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\rightarrow x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

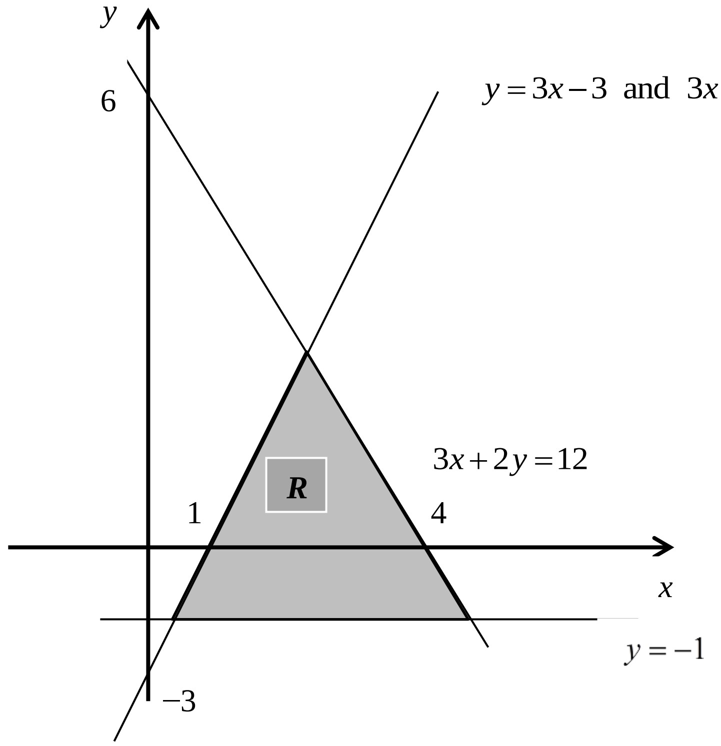
Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question number	Scheme	Marks
1(a)	Completes the square to find, $\mathrm{f}(x) = -2\left(x - \frac{5}{4}\right)^2 + \frac{73}{8}$ $p = -2 \quad q = -\frac{5}{4} \quad r = \frac{73}{8}$	M1 A2,1,0 (3)
	ALT $6 + 5x - 2x^2 = px^2 + 2pqx + pq^2 + r$ $\Rightarrow p = -2$ $-4q = 5 \Rightarrow q = -\frac{5}{4}$ $(-2)\left(\frac{25}{16}\right) + r = 6 \Rightarrow r = \frac{73}{8}$	 M1 A1 A1 (3)
(b)	(i) $\mathrm{f}(x) = \frac{73}{8}$ (ii) $x = \frac{5}{4}$	B1ft B1ft (2)
(c) (i)	$\mathrm{g}(x) = \frac{73}{8}$	M1A1
(ii)	$x^3 - \frac{5}{4} = 0 \Rightarrow x = \sqrt[3]{\frac{5}{4}}$	B1ft (3)
		[8]

Question number	Scheme	Marks
2 (a)		B1 B1 (2)

Question number	Scheme	Marks
(b)	Correct line drawn $y = -1$	B1
	Correct region shaded	B1 (2)
(c)	see table below	M1A1 M1A1 (4) [8]

Vertex	$(2,3)$	$\left(\frac{14}{3}, -1\right)$	$(\frac{2}{3}, -1)$
$P = 4x - y$	5	$\frac{59}{3}$	$\frac{11}{3}$
		greatest	

	Scheme	Marks
3	$\left(\dfrac{\mathrm{d}V}{\mathrm{d}t}=27\right)$	B1
	$r=\dfrac{3h}{2}$	
	$V=\dfrac{1}{3}\pi r^2 h \Rightarrow V=\dfrac{3}{4}\pi h^3$	M1A1
	$\dfrac{\mathrm{d}V}{\mathrm{d}h}=\dfrac{9}{4}\pi h^2$	M1
	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}t}\times\dfrac{\mathrm{d}h}{\mathrm{d}V}$	
	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=27\times\dfrac{4}{9\pi h^2}=27\times\dfrac{4}{9\pi 4^2}=0.23873...\ \dfrac{\mathrm{d}h}{\mathrm{d}t}=0.239$	M1dd A1
		[6]

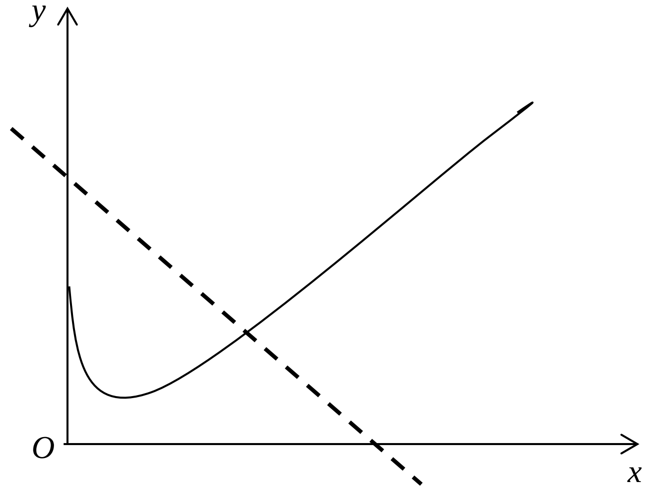
Question number	Scheme	Marks
4 (a)	When P is at rest $v = 0$ $2t^2 - 16t + 30 = 0 \Rightarrow (2t-6)(t-5) = 0$ $t = 3, 5$	M1A1 (2)
(b)	$\frac{dv}{dt} = 4t - 16$ $t = 3 \quad \frac{dv}{dt} = -4$ $t = 5 \quad \frac{dv}{dt} = 4$	M1 M1 A1 (3)
(c)	$s = \int (2t^2 - 16t + 30)\,dt = \frac{2t^3}{3} - 8t^2 + 30t \ (+c)$ when $t = 0$, $s = -4 \Rightarrow c = -4$ $s = \frac{2 \times 3^3}{3} - 8 \times 3^2 + 30 \times 3 - 4 = 32$ (m)	M1 B1 A1 (3) [8]

Question number	Scheme	Marks
5 (a)		

x	0	0.5	1	1.5	2	3	4
y	**2(.00)**	1.42	**1.5(0)**	2.15	**3.33**	7.25	**13.2(0)**

B1B1
(2)

(b)



B1ftB1ft
(2)

(c)

$$\frac{x^3+2}{x+1}=ax+b$$

$x^3+2=(x+1)(ax+b)=ax^2+x(a+b)+b$

$x^3-ax^2-x(a+b)+b-2\equiv x^3+x^2-3x-2$

$\Rightarrow a=-1$ and $b=4$ so required line is $y=-x+4$

M1A1A1

ALT

$x^3+x^2-3x-2=0\Rightarrow x^3+2=-x^2+3x+4\Rightarrow x^3+2=(x+1)(4-x)$

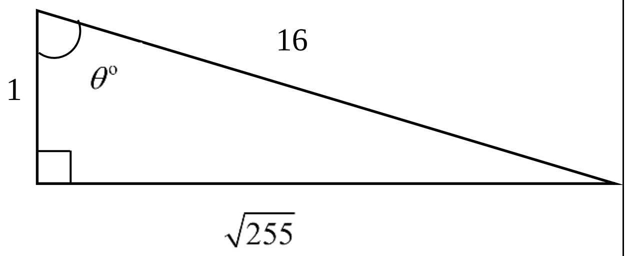
{M1A1A1}

$\Rightarrow 4-x=\dfrac{x^3+2}{x+1}\Rightarrow$ line required is $y=4-x$

Line $y=4-x$ drawn $\Rightarrow x=1.6$

M1A1
(5)

[9]

Question number	Scheme	Marks
6 (a)	$\tan\theta^{\circ} = \sqrt{255}$ $1^2 + 255 = 256$ $\sqrt{256} = 16$  $\Rightarrow \cos\theta^{\circ} = \dfrac{1}{16}$ *	M1A1cso (2)
(b)	$\cos\theta^{\circ} = \dfrac{1}{16} = \dfrac{x^2 + (x+4)^2 - (2x-2)^2}{2 \times x \times (x+4)}$ $\Rightarrow 0 = 17x^2 - 124x - 96$ $\Rightarrow x = \dfrac{124 \pm \sqrt{124^2 - 4 \times 17 \times (-96)}}{2 \times 17} = 8$ (other root not needed)	M1A1A1 M1A1 (5)
(c)	**Method 1** $\{AB = 8, \quad AC = 12, \quad BC = 14\}$ Uses sine rule to find ABC $\left[\theta^{\circ} = \tan^{-1}\sqrt{255} = 86.416...\right]$ $\dfrac{\sin 86.416}{14} = \dfrac{\sin ABC}{12} \Rightarrow \angle ABC = \sin^{-1} 0.855467... = 58.8^{\circ}$	M1A1 (2)
	Method 2 $\{AB = 8, \quad AC = 12, \quad BC = 14\}$ Uses cosine rule $\cos ABC = \dfrac{8^2 + 14^2 - 12^2}{2 \times 8 \times 14} = 0.5178... \Rightarrow ABC = 58.8^{\circ}$	{M1A1} {(2)}
(d)	Area = $\dfrac{1}{2} \times 8 \times 14 \times \sin 58.8 = 47.9$ (cm^2) **ALT** Uses Heron's formula $s = \dfrac{8+12+14}{2} = 17$ $A = \sqrt{17(17-8)(17-12)(17-14)} = 47.9$	M1A1 (2) {M1A1} [11]

Question number	Scheme	Marks
7 (a)	$(1-4x^2)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(-4x^2) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-4x^2)^2}{2!} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(-4x^2)^3}{3!}$ $(1-4x^2)^{-\frac{1}{2}} = 1 + 2x^2 + 6x^4 + 20x^6 + \ldots$	M1A1A1 (3)
(b)	$-\frac{1}{2} < x < \frac{1}{2}$ or $\lvert x\rvert < \frac{1}{2}$	B1 (1)
(c)	$(3+x)(1+2x^2+6x^4) = 3 + x + 6x^2 + 2x^3 + 18x^4$	M1M1A1 (3)
(d)	$\int_0^{0.3} \frac{3+x}{\sqrt{(1-4x^2)}}\,dx = \left[3x + \frac{x^2}{2} + 2x^3 + \frac{x^4}{2} + \frac{18x^5}{5}\right]_0^{0.3} = 1.011798 \approx 1.01$ (3sf)	M1A1M1d A1 (4) [11]

Question number	Scheme	Marks
8(a)	$\frac{ar^5}{ar} = 4 \Rightarrow r^4 = 4 \Rightarrow r = \pm\sqrt{2}$	M1A1 (2)
(b)	$ar^2 + ar^6 = 30 \Rightarrow a\left(r^2 + r^6\right) = 30$ $a\left[\left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^6\right] = 30 \Rightarrow 10a = 30 \Rightarrow a = 3$	M1A1A1 (3)
(c)	$S_{10} = \frac{3\left(\left(\sqrt{2}\right)^{10} - 1\right)}{\sqrt{2} - 1} = \left\{\frac{93}{\sqrt{2} - 1}\right\}$ or awrt 224.5 or $93\left(\sqrt{2} + 1\right)$	M1A1 (2)
(d)	$2400 < 3 \times \left(\sqrt{2}\right)^{(n-1)} \Rightarrow \left(\sqrt{2}\right)^{(n-1)} > 800$ $n - 1 > \log_{\sqrt{2}} 800 \Rightarrow n - 1 > 19.287... \Rightarrow n > 20.287...$ $n = 21$	M1 M1dA1 (3) [10]

Question number	Scheme	Marks
9 (a)	$x^2 - \text{sum} \times x + \text{product} = 0$ $x^2 + \frac{5}{2}x - 5 = 0$ $2x^2 + 5x - 10 = 0$ or integer multiples	M1A1 (2)
(b) (i)	$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \left(\frac{25}{4}\right) + 10 = \frac{65}{4}$ $(\alpha+\beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$	M1A1
(ii)	$\Rightarrow \alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = -\frac{125}{8} + 15\left(-\frac{5}{2}\right) = -\frac{425}{8}$ **ALT** $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2) = \left(-\frac{5}{2}\right)\left(\frac{73}{4} + 5\right) = -\frac{425}{8}$	M1A1A1 (5) {M1A1A1}
(c)	Product $\left(\alpha - \frac{1}{\alpha^2}\right) \times \left(\beta - \frac{1}{\beta^2}\right) = \left(\frac{\alpha^3 - 1}{\alpha^2}\right)\left(\frac{\beta^3 - 1}{\beta^2}\right) = \frac{\alpha^3\beta^3 - (\alpha^3 + \beta^3) + 1}{\alpha^2\beta^2}$ $= \frac{-125 - -\frac{425}{8} + 1}{36} = -\frac{567}{200}$ Sum $\left(\alpha - \frac{1}{\alpha^2}\right) + \left(\beta - \frac{1}{\beta^2}\right) = \left(\frac{\alpha^3 - 1}{\alpha^2}\right) + \left(\frac{\beta^3 - 1}{\beta^2}\right)$ $= \frac{\alpha^3\beta^2 - \beta^2 + \alpha^2\beta^3 - \alpha^2}{\alpha^2\beta^2} = \frac{\alpha^2\beta^2(\alpha+\beta) - (\alpha^2 + \beta^2)}{\alpha^2\beta^2}$ $= \frac{25\left(-\frac{5}{2}\right) - \frac{65}{4}}{25} = -\frac{63}{20}$ oe Equation Sum $= -\frac{63}{20}$, Product $= -\frac{567}{200}$ $\Rightarrow x^2 + \frac{63}{20}x - \frac{567}{200}\ (=0)$ $x^2 + \frac{314}{100}x - \frac{567}{200}\ (=0)$ M1 $200x^2 + 630x - 567 = 0$ A1	 M1 A1 M1 A1 M1A1 (6) [13]

Question number	Scheme	Marks
10 (a)	$\cos 2\theta = \cos^2\theta - \sin^2\theta \Rightarrow \cos 2\theta = \cos^2\theta - \left(1-\cos^2\theta\right)$	M1M1
	$\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow \cos^2\theta = \frac{1}{2}\left(\cos 2\theta + 1\right)$ *	A1cso (3)
(b)	(Uses $\cos^2\theta + \sin^2\theta = 1$ to give) $\cos 2\theta = 1 - 2\sin^2\theta$ seen anywhere	B1
	$4\cos^4\theta = \cos^2 2\theta + 2\cos 2\theta + 1 \Rightarrow$	
	$4\cos^4\theta = \frac{1}{2}\left(\cos 4\theta + 1\right) + 2\left(1 - 2\sin^2\theta\right) + 1 \Rightarrow$	M1 M1
	$8\cos^4\theta = \cos 4\theta + 1 + 4 - 8\sin^2\theta + 2 \Rightarrow$ $\cos 4\theta = 8\cos^4\theta + 8\sin^2\theta - 7$ *	M1 A1cso (5)
(c)	$16\cos^4\left(\theta - \frac{\pi}{6}\right) + 16\sin^2\left(\theta - \frac{\pi}{6}\right) - 15 = 0$	M1A1
	$\Rightarrow 8\cos^4\left(\theta - \frac{\pi}{6}\right) + 8\sin^2\left(\theta - \frac{\pi}{6}\right) - 7 = \frac{1}{2}$	M1A1 (4)
	$\cos 4\left(\theta - \frac{\pi}{6}\right) = \frac{1}{2} \Rightarrow 4\left(\theta - \frac{\pi}{6}\right) = \pm\frac{\pi}{3}, \pm\frac{5\pi}{3} \Rightarrow \theta = \frac{\pi}{4}$ or decimal equivalents awrt 0.785	
(d)	$\int_0^{\frac{\pi}{2}} \left(8\cos^4\theta + 8\sin^2\theta + 2\sin 2\theta\right)\,\mathrm{d}\theta = \int_0^{\frac{\pi}{2}} \left(\cos 4\theta + 2\sin 2\theta + 7\right)\,\mathrm{d}\theta$	M1M1M1dd A1 (4)
	$\Rightarrow = \left[\frac{\sin 4\theta}{4} - \cos 2\theta + 7\theta\right]_0^{\frac{\pi}{2}} = \left[\left(0 - (-1) + \frac{7\pi}{2}\right) - \left(0 - 1 + 0\right)\right] = 2 + \frac{7}{2}\pi$	
		[16]